2023年5月23日YII数学　SAT演習

●3－I　Numbers,Symbols, and Variables

Multiple-Choice

1. If $a = 9 \times 23$ and $b = 9 \times 124$, what is the value of $b - a$?
   (A) 901
   (B) 903
   (C) 906
   (D) 909
   (E) 911

2. By how much does the product of 8 and 25 exceed the product of 15 and 10?
   (A) 25
   (B) 50
   (C) 75
   (D) 100
   (E) 125

3. How many containers, each holding 16 fluid ounces of milk, are needed to hold 5 quarts of milk? (1 quart = 32 fluid ounces)
   (A) 6
   (B) 8
   (C) 10
   (D) 12
   (E) 14

4. If the current odometer reading of a car is 31,983 miles, what is the LEAST number of miles that the car must travel before the odometer displays four digits that are the same?
   (A) 17
   (B) 239
   (C) 350
   (D) 650
   (E) 1,350

5. In store A a scarf costs \$12, and in store B the same scarf is on sale for \$8. How many scarfs can be bought in store B with the amount of money, excluding sales tax, needed to buy 10 scarfs in store A?
   (A) 4
   (B) 12
   (C) 15
   (D) 18
   (E) 21

6. Let * represent one of the four basic arithmetic operations such that, for any nonzero real number r,

   $$r * 0 = r \text{ and } r * r = 0$$

   Which arithmetic operation(s) does the symbol * represent?
   (A) + only
   (B) − only
   (C) + and −
   (D) × only
   (E) +, −, or ×

7. Kurt has saved \$160 to buy a stereo system that costs \$400 including taxes. If he earns \$8 an hour after all payroll deductions have been made, how many hours will he have to work in order to have exactly enough money to buy the stereo system?
   (A) 20
   (B) 24
   (C) 25
   (D) 30
   (E) 40

8. If the present time is exactly 1:00 P.M., what was the time exactly 39 hours ago?

(A) 4:00 P.M.
(B) 4:00 A.M.
(C) 9:00 P.M.
(D) 9:00 A.M.
(E) 10:00 P.M.

9. Let # represent one of the four basic arithmetic operations such that, for any nonzero real numbers r and s,

$$r \# 0 = r \quad \text{and} \quad r \# s = s \# r$$

Which arithmetic operation(s) does the symbol # represent?

(A) + only
(B) × only
(C) − only
(D) − and ×
(E) + and ÷

10. If $w = (6)(6)(6)$, $x = (5)(6)(7)$, and $y = (4)(6)(8)$, which inequality statement is true?

(A) $x < y < w$
(B) $w < x < y$
(C) $y < w < x$
(D) $y < x < w$
(E) $w < y < x$

11. If x and y are positive integers, $2x + y < 29$, and $y > 4$, what is the greatest possible value of $x - y$?

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

12.
$$r = 4pv$$
$$q = \frac{r}{p + 2}$$

If r and q are defined by the equations above, what is the value of q when $p = 3$ and $v = 5$?

(A) 4
(B) 6
(C) 8
(D) 12
(E) 15

Grid-In

1. The houses in a certain community are numbered consecutively from 2019 to 2176. How many houses are in the community?

2. If 1 kilobyte of computer memory is equivalent to 1024 bytes and 1 byte is equivalent to 8 bits, how many kilobytes are equivalent to 40,960 bits?

3. A television set will cost, including taxes and finance charges, \$495 if the buyer puts \$129 down and then pays off the balance in eight equal monthly payments. Under this purchase plan, what will each of the monthly payments be?

4. If $13 \leq k \leq 21$, $9 \leq p \leq 19$, $2 < m < 6$, and k, p, and m are integers, what is the largest possible value of $\frac{k - p}{m}$?

5. For some fixed value of x, $9(x + 2) = y$. After the value of x is increased by 3, $9(x + 2) = w$. What is the value of $w - y$?

6. If x and y are positive integers, and $3x + 2y = 21$, what is the sum of all possible values of x?

●3-2 Powers and Roots その１

Multiple-Choice

1. If $5 = a^x$, then $\frac{5}{a} =$

(A) a^{x+1}
(B) a^{x-1}
(C) a^{1-x}
(D) $a^{\frac{x}{5}}$
(E) $a^{\frac{5}{x}}$

2. What is the greatest number of positive integer values of x for which $1 < x^2 < 50$?

(A) Five
(B) Six
(C) Seven
(D) Eight
(E) Nine

3. If k is a positive integer, which of the following is NOT equivalent to $(2^{3k})^2$?

(A) $(2^k)^6$
(B) 64^k
(C) $4^k(2^{4k})$
(D) $(8^k)^3$
(E) $2^{3k}(2^{3k})$

4. If $\frac{x^{23}}{x^m} = x^{15}$ and $(x^4)^n = x^{20}$, then $mn =$

(A) 13
(B) 24
(C) 28
(D) 32
(E) 40

5. If $2 = p^3$, then $8p$ must equal

(A) p^6
(B) p^8
(C) p^{10}
(D) $8\sqrt{2}$
(E) 16

6. If $b^3 = 4$, then $b^6 =$

(A) 2
(B) 8
(C) 12
(D) 16
(E) 64

7. If w is a positive number and $w^2 = 2$, then $w^3 =$

(A) $\sqrt{2}$
(B) $2\sqrt{2}$
(C) 4
(D) $3\sqrt{2}$
(E) 6

8. If $2^x = y^2$, which of the following must be equal to 2^{x+1}?

(A) y^3
(B) $y^2 + 1$
(C) $2y^2$
(D) $4y^2$
(E) $\frac{y^2}{2}$

9. If x is a positive integer such that $x^9 = r$ and $x^5 = w$, which of the following must be equal to x^{13}?

(A) $rw - 1$
(B) $r + w - 1$
(C) $\frac{r^2}{w}$
(D) $r^2 - w$
(E) $\frac{r}{3} + 2w$

10. Given $y = wx^2$ and y is not 0. If the values of x and w are each doubled, then the value of y is multiplied by

(A) 1
(B) 2
(C) 4
(D) 6
(E) 8

11. If $\sqrt{n}$ is a positive integer, how many values of n are in the interval $100 < n < 199$?

(A) Three
(B) Four
(C) Five
(D) Six
(E) Seven

12. If $(2^3)^2 = 4^p$, then $3^p =$

(A) 3
(B) 6
(C) 9
(D) 27
(E) 81

13. If a and b are positive integers, then $3^{a+b} \cdot 6^a =$

(A) 18^{2a+b}
(B) $18^{a(a+b)}$
(C) $3^{6(2a+b)}$
(D) $9^{a+b} \cdot 2^a$
(E) $3^{2a+b} \cdot 2^a$

14. If x is a positive integer, which of the following statements must be true?

I. $\left(\frac{x}{x}\right)^{99} = \left(\frac{x+1}{x+1}\right)^{100}$

II. $(x^x)^2 = x^{x^2}$

III. $\left(\frac{x^{100}}{x^{99}}\right) = 1^x$

(A) I only
(B) II only
(C) I and III
(D) II and III
(E) I, II, and III

15. If $y = 25 - x^2$ and $1 \leq x \leq 5$, what is the smallest possible value of y?

(A) 0
(B) 1
(C) 5
(D) 10
(E) 15

16. If $x = \sqrt{6}$ and $y^2 = 12$, then $\frac{4}{xy} =$

(A) $\frac{3}{2\sqrt{2}}$
(B) $\frac{\sqrt{2}}{3}$
(C) $\frac{3}{\sqrt{2}}$
(D) $\frac{2\sqrt{2}}{3}$
(E) $\frac{\sqrt{6}}{3}$

Grid-In

1. If $2^4 \times 4^2 = 16^x$, then $x =$

2. If $a^7 = 7777$ and $\frac{a^6}{b} = 11$, what is the value of ab?

3. If $(y - 1)^3 = 8$, what is the value of $(y + 1)^2$?

4. If $\frac{p + p + p}{p \cdot p \cdot p} = 12$ and $p > 0$, what is the value of p?

●3-3 Powers and Roots その２

Multiple-Choice

1. Which number has the most factors?

(A) 12
(B) 18
(C) 25
(D) 70
(E) 100

2. When a whole number N is divided by 5, the quotient is 13 and the remainder is 4. What is the value of N?

(A) 55
(B) 59
(C) 65
(D) 69
(E) 79

3. If p is divisible by 3 and q is divisible by 4, then pq must be divisible by each of the following EXCEPT

(A) 3
(B) 4
(C) 6
(D) 9
(E) 12

4. If the sum of the factors of 18 is S and the sum of prime numbers less than 18 is P, then P exceeds S by what number?

(A) 19
(B) 17
(C) 15
(D) 13
(E) 11

5. Which number is divisible by 2 and by 3?

(A) 112
(B) 4,308
(C) 6,122
(D) 23,451
(E) 701,456

6. All numbers that are divisible by both 3 and 10 are also divisible by

(A) 4
(B) 9
(C) 12
(D) 15
(E) 20

7. If x represents any even number and y represents any odd number, which of the following numbers is even?

(A) $y + 2$
(B) $x - 1$
(C) $(x + 1)(y - 1)$
(D) $y(y + 2)$
(E) $x + y$

8. For how many different positive integers p is $\frac{105}{p}$ also an integer?

(A) Five
(B) Six
(C) Seven
(D) Eight
(E) Nine

9. If n is an odd integer, which expression always represents an odd integer?

(A) $(2n - 1)^2$
(B) $n^2 + 2n + 1$
(C) $(n - 1)^2$
(D) $\frac{n + 1}{2}$
(E) $3^n + 1$

10. If $k - 1$ is a multiple of 4, what is the next larger multiple of 4?

(A) $k + 1$
(B) $4k$
(C) $k - 5$
(D) $k + 3$
(E) $4(k - 1)$

11. After m marbles are put into n jars, each jar contains the same number of marbles, with two marbles remaining. In terms of m and n, how many marbles were put into each jar?

(A) $\frac{m}{n} + 2$
(B) $\frac{m}{n} - 2$
(C) $\frac{m + 2}{n}$
(D) $\frac{m - 2}{n}$
(E) $\frac{mn}{n + 2}$

12. When p is divided by 4, the remainder is 3; and when p is divided by 3, the remainder is 0. What is a possible value of p?

(A) 8
(B) 11
(C) 15
(D) 18
(E) 21

13. If n is an integer and $n^2 + 5$ is an odd integer, then which statement(s) must be true?

I. $n^2 - 1$ is even.
II. n is even.
III. $5n$ is even.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

14. When the number of people who contribute equally to a gift decreases from four to three, each person must pay an additional \$10. What is the cost of the gift?

(A) \$30
(B) \$60
(C) \$90
(D) \$120
(E) \$180

15. If n is any even integer, what is the remainder when $(n + 1)^2$ is divided by 4?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

16. A jar contains between 40 and 50 marbles. If the marbles are taken out of the jar three at a time, two marbles will be left in the jar. If the marbles are taken out of the jar five at a time, four marbles will be left in the jar. How many marbles are in the jar?

(A) 41
(B) 43
(C) 44
(D) 47
(E) 49

Grid-In

1. When a is divided by 7, the remainder is 5; and when b is divided by 7, the remainder is 4. What is the remainder when $a + b$ is divided by 7?

2. How many integers from $-3{,}000$ to 3,000, inclusive, are divisible by 3?

3. When a positive integer k is divided by 6, the remainder is 1. What is the remainder when $5k$ is divided by 3?

4. As computer circuit boards move along an assembly production line in single file, a quality-control inspector checks every third circuit board beginning with the third board. A second quality-control inspector checks every fifth circuit board beginning with the fifth board. If 100 computer circuit boards were produced on the assembly line while both inspectors were working, how many of these boards were NOT checked by either inspector?

●3-4 Number Lines and signed Numbers

Multiple-Choice

1. If $2b = -3$, what is the value of $1 - 4b$?

   (A) -7
   (B) -5
   (C) 5
   (D) 6
   (E) 7

2. If a is a negative integer and b is a positive integer, which of the following statements must be true?

   I. $b + a > 0$
   II. $\frac{b - a}{a} < 0$
   III. $a^b < 0$

   (A) None
   (B) I only
   (C) II only
   (D) III only
   (E) I and II only

3. If the product of five numbers is positive, then, at most, how many of the five numbers could be negative?

   (A) One
   (B) Two
   (C) Three
   (D) Four
   (E) Five

4. For which value of k is the value of $k(k - 2)(k + 1)$ negative?

   (A) -2
   (B) -1
   (C) 0
   (D) 2
   (E) 3

5. $p^2(2 - 5) + (-p)^2 =$

   (A) $-4p^2$
   (B) $-p^2$
   (C) $-2p^2$
   (D) $2p^2$
   (E) $4p^2$

6. If $n + 5$ is an odd integer, then n could be which of the following?

   (A) 1
   (B) -1
   (C) -2
   (D) -3
   (E) -7

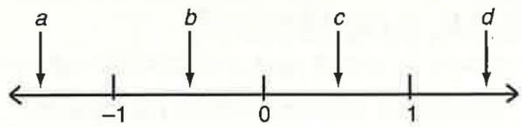
7. Which of the following statements must be true when $a < 0$ and $b > 0$?

   I. $a + b > 0$
   II. $b - a > 0$
   III. $a\left(\frac{a}{b}\right) > 0$

   (A) I only
   (B) II only
   (C) I and III only
   (D) II and III only
   (E) I, II, and III

8. $(-2)^3 + (-3)^2 =$

   (A) -12
   (B) -1
   (C) 0
   (D) 1
   (E) 2



<u>Questions 9 and 10</u> refer to the diagram above.

9. Which of the following statements must be true?

   I. $c^2 < c$
   II. $a^2 > c$
   III. $b < \frac{1}{b}$

   (A) I only
   (B) I and II only
   (C) I and III only
   (D) I, II, and III
   (E) None

10. Which of the following statements must be true?

    I. $ad > b$
    II. $ab > ad$
    III. $\frac{1}{a} > \frac{1}{b}$

    (A) II only
    (B) I and II only
    (C) II and III only
    (D) I, II, and III
    (E) None

11. If $(y - 3)^2 = 16$, what is the smallest possible value of y^2?

    (A) -4
    (B) 1
    (C) 7
    (D) 16
    (E) 49

12. If X represents the sum of the 10 greatest negative integers and Y represents the sum of the 10 least positive integers, which of the following must be true?

    I. $X + Y < 0$
    II. $Y - X = 2Y$
    III. $X^2 = Y^2$

    (A) None
    (B) I only
    (C) III only
    (D) I and II only
    (E) II and III only

13. If $a^2b^3c > 0$, which of the following statements must be true?

    I. $bc > 0$
    II. $ac > 0$
    III. $ab > 0$

    (A) I only
    (B) I and II only
    (C) I and III only
    (D) II and III only
    (E) I, II, and III

14. If $a^2b^3c^5$ is negative, which product is always negative?

    (A) bc
    (B) b^2c
    (C) ac
    (D) ab
    (E) bc^2

15. If $a \neq 0$, which of the following statements must be true?

    I. $(-a)^2 = a^2 - 2a^2$
    II. $a - b = -(b - a)$
    III. $a > -a$

    (A) I only
    (B) II only
    (C) III only
    (D) I and III only
    (E) II and III only

Grid-In

1. If $p^2 = 16$ and $q^2 = 36$, what is the largest possible value of $q - p$?

2. If $-4 \leq x \leq 2$ and $y = 1 - x^2$, what number is obtained when the smallest possible value of y is subtracted from the *largest* possible value of y?

●解答・解説

Lesson 3-1

MULTIPLE-CHOICE

1. **(D)** If $a = 9 \times 23$ and $b = 9 \times 124$, then
$$b - a = 9 \times 124 - 9 \times 23$$
You can use your calculator to do the arithmetic on the right side of the equation, but it's easier to use the reverse of the distributive law:
$$\begin{aligned} b - a &= 9(124 - 23) \\ &= 9(101) \\ &= 909 \end{aligned}$$
2. **(B)** To figure out by what amount quantity A exceeds quantity B, calculate $A - B$: $(8 \times 25) - (15 \times 10) = 200 - 150 = 50$
3. **(C)** Since 1 quart = 32 fluid ounces, 5 quarts = $5 \times 32 = 160$ fluid ounces. If each container holds 16 fluid ounces, then $\frac{160}{16} = 10$ containers are needed to hold 5 quarts of milk.
4. **(B)** *Solution 1*: If the current odometer reading of a car is 31,983 miles, then the next mileage reading in which at least four digits are the same will be 32,222. Hence, the least number of miles that the car must travel before the odometer displays four digits that are the same is $32{,}222 - 31{,}983 = 239$.
Solution 2: Find the sum of 31,983 and the number given in each answer choice beginning with (A). Stop when you get a sum (31,983 + 239) in which at least four of the digits are the same. Hence, the correct choice is (B).
5. **(C)** In store A 10 scarfs cost $10 \times \$12 = \$120$. Since the same scarf costs \$8 in store B, $\frac{120}{8} = 15$ scarfs can be bought in store B with \$120.
6. **(B)** If $r * 0 = r$, then $*$ represents either addition or subtraction or both. It is also given that $r * r = 0$. Since $r - r = 0$ and $r + r \neq 0$ when r is not 0, $*$ can represent only subtraction.
7. **(D)** Since Kurt has saved \$160 to buy a stereo system that costs \$400, he needs to earn an additional $\$400 - \$160 = \$240$. Earning \$8 an hour, he will need to work $\frac{240}{8} = 30$ hours to have enough money to buy the stereo system.
8. **(E)** Since 24 hours + 12 hours + 3 hours = 39 hours, break down the problem by figuring out the time 24 hours ago, 12 hours before that time, and then 3 hours earlier. If the present time is exactly 1:00 P.M., then 24 hours ago it was also 1:00 P.M., and 12 hours before that time it was 1:00 A.M. Three hours before 1:00 A.M. the time was 10:00 P.M.
9. **(A)** If $r \# 0 = r$, then the symbol # can represent either addition or subtraction, but not multiplication or division. Since it is also given that $r \# s = s \# r$, the operation # is commutative. Since addition is commutative and subtraction is not commutative, the symbol # represents only addition.
10. **(D)** Each product contains 6, so 6 can be ignored. Then $w = 36$, $x = 35$, and $y = 32$, so $y < x < w$.
11. **(B)** If $2x + y < 29$, the expression $x - y$ will have its greatest possible value when y has its least possible value and x has its maximum value.
    - Since y is an integer and $y > 4$, the least possible value for y is 5.
    - If $y = 5$, then $2x + y < 29$ becomes $2x + 5 < 29$ so $2x < 24$ and $x < 12$.
    - Since x is an integer and $x < 12$, the maximum value of x is 11.
    - Hence, $11 - 5 = 6$ is the greatest possible value of $x - y$.
12. **(D)** If r and q are defined by $r = 4pv$ and $q = \frac{r}{p+2}$, then when $p = 3$ and $v = 5$:
$r = 4pv = 4 \times 3 \times 5 = 60$ and
$$\begin{aligned} q &= \frac{r}{p+2} \\ &= \frac{60}{3+2} \\ &= \frac{60}{5} \\ &= 12 \end{aligned}$$

GRID-IN

1. (**158**) In general, if A and B are positive integers, then the number of integers from A to B is $(B - A) + 1$. If the number of houses in a certain community are numbered consecutively from 2,019 to 2,176, there are $(2{,}176 - 2{,}019) + 1 = 157 + 1 = 158$ houses in the community.
2. (**5**) Since 1 kilobyte is equivalent to $1{,}024 \times 8$ or 8,192 bits, 40,960 bits are equivalent to $\frac{40{,}960}{8{,}192}$ or 5 kilobytes.
3. (**45.8**) The balance that needs to be paid off is \$495 − \$129 or \$366. Since eight equal monthly payments will be made, each monthly payment is $\frac{\$366}{8}$ or \$45.75. Since 45.75 will not fit the grid, grid in 45.8.
4. (**4**) The fraction will have its largest value when $\frac{k-p}{m}$ has its greatest value and m has its least value. The largest value of $k - p$ is $21 - 9$ or 12. The inequality $2 < m < 6$ means that m is greater than 2 but less than 6. Since m is an integer, the least value of m is 3. Hence, the largest possible value of $\frac{k-p}{m}$ is $\frac{12}{3}$ or 4.
5. (**27**) For some fixed value of x, $9(x + 2) = y$. If the value of x is increased by 3, then the value of y is increased by $9 \times 3 = 27$. Since after x is increased by 3, $9(x + 2) = w$, the value of $w - y$ is 27.
6. (**9**) To find the sum of all possible values of x given $3x + 2y = 21$, where x and y are positive integers, plug successive integer values for x starting with 1 into the given equation and note which values of x produce positive integer values for y.

x	$y = \frac{21 - 3x}{2}$
1	$y = \frac{21 - 3}{2} = \frac{18}{2} = 9$
2	$y = \frac{21 - 3(2)}{2} = \frac{15}{2}$
3	$y = \frac{21 - 3(3)}{2} = \frac{12}{2} = 6$
4	$y = \frac{21 - 3(4)}{2} = \frac{9}{2}$
5	$y = \frac{21 - 3(5)}{2} = \frac{6}{2} = 3$
6	$y = \frac{21 - 3(6)}{2} = \frac{3}{2}$
7	$y = \frac{21 - 3(7)}{2} = \frac{0}{2}$ Stop!

Any value of x greater than 7 will produce a negative value for y. Since $x = 1$, 3, and 5 produce positive integer values for y, the sum of all possible values of x is $1 + 3 + 5 = 9$.

Lesson 3-2

MULTIPLE-CHOICE

1. (**B**) To divide powers with the *same* base, keep the base and *subtract* the exponents. If $5 = a^x$, then

$$\frac{5}{a} = \frac{a^x}{a}$$
$$= a^{x-1}$$

2. (**B**) If $1 < x^2 < 50$, then $\sqrt{1} < \sqrt{x^2} < \sqrt{50}$, which can be written as $1 < x < \sqrt{50}$. Since $\sqrt{50}$ is between 7 and 8, there are six integer values of x that are greater than 1 but less than $\sqrt{50}$: 2, 3, 4, 5, 6, and 7.
3. (**D**) The given expression, $(2^{3k})^2$, is equivalent to $2^{3k \cdot 2}$ or 2^{6k}. Using the laws of exponents for positive integers, rewrite each of the answer choices as a power of 2:
   - (A) $(2^k)^6 = 2^{6k}$ ✓
   - (B) $64^k = (2^6)^k = 2^{6k}$ ✓

- (C) $4^k(2^{4k}) = (2^2)^k \cdot (2^{4k}) = (2^{2k}) \cdot (2^{4k}) = 2^{2k+4k} = 2^{6k}$ ✓
- (D) $(8^k)^3 = (2^{3k})^3 = 2^{3k \cdot 3} = 2^{9k}$ ✗
- (E) $2^{3k}(2^{3k}) = 2^{3k+3k} = 2^{6k}$ ✓

4. **(E)** Since $\frac{x^{23}}{x^m} = x^{15}$, $x^m = \frac{x^{23}}{x^{15}} = x^{23-15} = x^8$, so $m = 8$. If $\left(x^4\right)^n = x^{20}$, then $4n = 20$, so $n = \frac{20}{4} = 5$. Thus, $mn = 8 \times 5 = 40$.
5. **(C)** If $2 = p^3$, then
$$\begin{aligned}(2)^3 &= (p^3)^3\\ 8 &= p^{3\times3}\\ &= p^9\end{aligned}$$
Hence, $8p = p^9 \times p = p^{10}$.
6. **(D)** If $b^3 = 4$, then
$b^6 = (b^3)^2 = (4)^2 = 16$
7. **(B)** If w is a positive number and $w^2 = 2$, then $w = \sqrt{2}$, so
$w^3 = w^2 \bullet w = 2\sqrt{2}$
8. **(C)** Break down 2^{x+1}: $2^{x+1} = 2^1 \cdot 2^x = 2y^2$.
9. **(C)** Test each answer choice in turn by replacing r with x^9 and w with x^5. Only choice (C) is true:
   - (A) $rw - 1 = x^9 \cdot x^5 - 1 = x^{9+5} - 1 = x^{14} - 1 \neq x^{13}$
   - (B) $r + w - 1 = x^9 + x^5 - 1 \neq x^{13}$
   - (C) $\frac{r^2}{w} = \frac{(x^9)^2}{x^5} = \frac{x^{18}}{x^5} = x^{18-5} = x^{13}$
   - (D) $r^2 - w = (x^9)^2 - x^5 = x^{18} - x^5 \neq x^{13}$
   - (E) $\frac{r}{3} + 2w = \frac{x^9}{3} + 2x^5 \neq x^{13}$
10. **(E)** Given $y = wx^2$ and y is not 0. Since the values of x and w are each doubled, replace w with $2w$ and x with $2x$ in the original equation:
$$\begin{aligned}y_{new} &= (2w)(2x)^2\\ &= (2w)(4x^2)\\ &= 8(wx^2)\\ &= 8y\end{aligned}$$
Hence, the original value of y is multiplied by 8.
11. **(B)** If $\sqrt{n}$ is a positive integer, then n must be a perfect square integer. The perfect square integers in the interval $100 < n < 199$ are as follows:
$121(=11^2)$, $144(=12^2)$, $169(=13^2)$, $196(=14^2)$
Hence, there are four perfect square integers in the given interval.
12. **(D)** If $(2^3)^2 = 4p$, you can find p by expressing each side of the equation as a power of the same base and then setting the exponents of the two bases equal:
$$\begin{aligned}(2^3)^2 &= 4^p\\ 2^6 &= 2^{2p}\\ 6 &= 2p\\ 3 &= p\end{aligned}$$
Since $p = 3$, then
$3^p = 3^3 = 3 \times 3 \times 3 = 27$
13. **(E)** Rewrite 6^a as $(3 \cdot 2)^a$. Then use the laws of exponents:
$$\begin{aligned}3^{a+b} \cdot 6^a &= 3^{a+b} \cdot (3 \cdot 2)^a\\ &= 3^{a+b} \cdot 3^a \cdot 2^a\\ &= (3^{a+b} \cdot 3^a) \cdot 2^a\\ &= 3^{2a+b} \cdot 2^a\end{aligned}$$
14. **(A)** Determine whether each Roman numeral statement is true or false when x is a positive integer.
   - I. Since $\left(\frac{x}{x}\right)^{99} = (1)^{99} = 1$ and $\left(\frac{x+1}{x+1}\right)^{100} = (1)^{100} = 1$, statement I is true.
   - II. Since $(x^x)^2 = x^{2x}$ and x^{x^2} is not equal to x^{2x} for all positive integer values of x, statement II is false.
   - III. Since $\frac{x^{100}}{x^{99}} = x^{100-99} = x^1 = x$ and $1^x = 1$, statement III is false.

   Since only Roman numeral statement I is true, the correct choice is (A).
15. **(A)** If $y = 25 - x^2$, the smallest possible value of y is obtained by subtracting the largest possible value of x^2 from 25. Since $1 \leq x \leq 5$, the largest possible value of x^2 is $5^2 = 25$. When $x^2 = 25$, then $y = 25 - 25 = 0$.

16. (**B**) If $x = \sqrt{6}$ and $y^2 = 12$, then $y = \sqrt{12}$, so

$$\frac{4}{xy} = \frac{4}{\sqrt{6}\sqrt{12}} = \frac{4}{\sqrt{72}} = \frac{4}{\sqrt{36}\sqrt{2}} = \frac{4}{6\sqrt{2}} = \frac{2}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\not{2}\sqrt{2}}{3\cdot\not{2}} = \frac{\sqrt{2}}{3}$$

GRID-IN

1. (**2**) Given $2^4 \times 4^2 = 16^x$, find the value of x by expressing each side of the equation as a power of the same base.

$$2^4 \times (2^2)^2 = (2^4)^x$$
$$2^4 \times 2^4 = 2^{4x}$$
$$2^{4+4} = 2^{4x}$$
$$2^8 = 2^{4x}$$
$$8 = 4x, \text{ so } x = 2$$

2. (**707**) Since $\frac{a^6}{b} = 11$, $a^6 = 11b$. Thus,

$$a^7 = a \times a^6 = 7777$$
$$a \times (11b) = 7777$$
$$11ab = 7777$$
$$\frac{11ab}{11} = \frac{7777}{11}$$
$$ab = 707$$

3. (**16**) Since $2^3 = 2 \times 2 \times 2 = 8$ and $(y - 1)^3 = 8$, then $y - 1 = 2$, so $y = 3$. Hence, $(y + 1)^2 = (3 + 1)^2 = 4^2 = 4 \times 4 = 16$

4. (**1/2**) Since

$$\frac{p + p + p}{p \cdot p \cdot p} = \frac{3p}{p \cdot p \cdot p} = \frac{3}{p^2} = 12$$

then $\frac{p^2}{3} = \frac{1}{12}$, so $p^2 = \frac{3}{12} = \frac{1}{4}$. Hence, $p = \frac{1}{2}$ since $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Grid in as 1/2.

Lesson 3-3

MULTIPLE-CHOICE

1. (**E**) When figuring out how many factors a number has, be sure to include the number itself and 1. Try each choice in turn:
   - (A) There are 5 factors of 12: 1, 3, 4, 6, and 12.
   - (B) There are 6 factors of 18: 1, 2, 3, 6, 9, and 18.
   - (C) There are 3 factors of 25: 1, 5, and 25.
   - (D) There are 8 factors of 70: 1, 2, 5, 7, 10, 14, 35, and 70.
   - (E) There are 9 factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, and 100.

   Hence, 100 has the most factors. The correct choice is (E).
2. (**D**) In any division example, the divisor times the quotient plus the remainder should equal the dividend. If the quotient of N divided by 5 is 13 and the remainder is 4, then $\frac{N}{5} = 13 + \frac{4}{5}$, so
   $N = (5 \times 13) + 4 = 65 + 4 = 69$
3. (**D**) *Solution 1*: If p is divisible by 3 and q is divisible by 4, then pq must be divisible by any combination of prime factors of 3 and 4. Since $3 = 3 \times 1$ and $4 = 2 \times 2$, pq is divisible by each of the following: 3, 4, 3×2 or 6, and $3 \times 2 \times 2$ or 12. Since no product of prime factors of 3 and 4 equals 9, pq cannot be divisible by 9.
   Solution 2: Pick numbers for p and q. Then test each choice until you find a number that does not divide pq evenly. For example, if $p = 6$ and $q = 8$, then $pq = 48$. Testing each answer choice, you find that 48 is divisible by 3, 4, and 6, but not by 9.
4. (**A**) Find the value of $P - S$:
   - If S represents the sum of the factors of 18, then
     $S = 1 + 2 + 3 + 6 + 9 + 18 = 39$
   - If P represents the sum of the prime numbers less than 18, then
     $P = 2 + 3 + 5 + 7 + 11 + 13 + 17 = 58$
   - Hence, $P - S = 58 - 39 = 19$.

5. **(B)** *Solution 1*: A number is divisible by 3 if the sum of its digits is divisible by 3. In choice (B), the sum of the digits of 4308 is $4 + 3 + 0 + 8 = 15$, which is divisible by 3. In choice (E), the sum of the digits of 23,451 is $2 + 3 + 4 + 5 + 1 = 15$, so (E) is also divisible by 3. A number is divisible by 2 only if its last digit is even. Since the last digit of 4308 is even but the last digit of 23,451 is odd, the correct choice must be (B).
   Solution 2: Using a calculator, test each choice in turn to find a number that gives a 0 remainder when divided by 2 and by 3.
6. **(D)** *Solution 1*: Since 3 and 10 do not have any common factors other than 1, any number that is divisible by both 3 and 10 must be divisible by their product, 30. Any number that is divisible by 30 must also be divisible by any factor of 30. Since 15 is the only answer choice that is a factor of 30, any number that is divisible by both 3 and 10 must also be divisible by 15.
   Solution 2: Pick an easy number that is divisible by both 3 and 10, say 30. Then divide 30 by each of the answer choices in turn. Stop when you find a number (15) that divides evenly into 30. The correct choice is (D).
7. **(C)** Since x represents any even number and y represents any odd number, let $x = 2$ and $y = 3$. Evaluate the expression in each of the answer choices until you find one that produces an even number.
   - (A) $y + 2 = 3 + 2 = 5$
   - (B) $x - 1 = 2 - 1 = 1$
   - (C) $(x + 1)(y - 1) = (2 + 1)(3 - 1) = (3)(2) = 6$. There is no need to go further. The correct choice is (C).
8. **(D)** Break down 105 into its prime factors:
   $$\frac{105}{p} = \frac{1 \times 5 \times 21}{p} = \frac{1 \times 3 \times 5 \times 7}{p}$$
   Thus, when p equals any of the eight positive integers 1, 3, 5, 7, 15 (3×5), 21 (3×7), 35 (5×7), or 105, $\frac{105}{p}$ is an integer.
9. **(A)** Since n is an odd integer, let $n = 3$. Evaluate each answer choice in turn until you find an odd number. For choice (A),
   $$(2n - 1)^2 = (2(3) - 1)^2 = (6-1)^2 = 25$$
   $$= 2(3) - 1 = 5$$
   There is no need to go further. The correct choice is (A).
10. **(D)** Consecutive multiples of 4, such as 4, 8, and 12, always differ by 4. If $k - 1$ is a multiple of 4, then the next larger multiple of 4 is obtained by adding 4 to $k - 1$, which gives $k - 1 + 4$ or $k + 3$.
11. **(D)** You are told that, after m marbles are put into n jars, each jar contains the same number of marbles, with two marbles remaining. If x represents the number of marbles put into each jar, m divided by n equals x with a remainder of 2. This statement can be written as
    $$\frac{m}{n} = x + \frac{2}{n}$$
    Since
    $$x = \frac{m}{n} - \frac{2}{n} = \frac{m - 2}{n},$$
    $\frac{m-2}{n}$ marbles were put into each jar.
12. **(C)** You are given that, when p is divided by 4, the remainder is 3, and when p is divided by 3, the remainder is 0. Identify the answer choices that are divisible by 3. Then substitute each of these choices for p until you find the choice that produces a remainder of 3 when divided by 4.
    Choices (C), (D), and (E) are each divisible by 3. For choice (C), let $p = 15$. When 15 is divided by 4, the remainder is 3.
    There is no need to go further. The correct choice is (C).
13. **(D)** Determine whether each Roman numeral statement is true or false.
    - I. Subtracting any multiple of 2 from an odd integer always produces another odd integer. For example, $7 - 4 = 3$. Since $n^2 + 5$ is an odd integer and $(n^2 + 5) - 6 = n^2 - 1$, $n^2 - 1$ is also an odd integer. Hence, statement I is false.

- II. The sum of an even integer and an odd integer is an odd integer. Since $n^2 + 5$ is an odd integer, n^2 must be even, so n must also be even. Hence, statement II is true.
- III. The product of an odd integer and an even integer is an even integer. Since you have determined that n is even, $5n$ is also even. Hence, statement III is true.

Since only Roman numeral statements II and III are true, the correct choice is (D).

14. **(D)** You are told that, if the number of people who contribute equally to a gift decreases from four to three, each person must pay an additional \$10. You can eliminate choices (A) and (C), which are not divisible by both 3 and 4. Check each of the remaining choices until you find the right one.
    - (B) 60 divided by 3 is 20, and 60 divided by 4 is 15. The difference between 20 and 15 is 5.
    - (D) 120 divided by 3 is 40, and 120 divided by 4 is 30. The difference between 40 and 30 is 10.

    There is no need to test the last choice. The correct choice is (D).
15. **(B)** Since n is any even integer, pick a simple number for n. If $n = 4$, then $(n+1)^2 = (4+1)^2 = 25$. When 25 is divided by 4, the remainder is 1.
16. **(C)** You need to find the answer choice that produces remainders of 2 and 4 when divided by 3 and 5, respectively.
    - (A) The remainder when 41 is divided by 3 is 2, and the remainder when 41 is divided by 5 is 1.
    - (B) The remainder when 43 is divided by 3 is 1.
    - (C) The remainder when 44 is divided by 3 is 2, and the remainder when 44 is divided by 5 is 4.

    There is no need to continue. The correct choice is (C).

GRID-IN

1. **(2)** When a is divided by 7, the remainder is 5, so let $a = 12$. When b is divided by 7, the remainder is 4, so let $b = 11$. Then $a + b = 23$. When 23 is divided by 7, the remainder is 2.
2. **(2,001)** All multiples of 3 from -3000 to 3000 are divisible by 3. Since

   $$3 = 1 \times 3, 6 = 2 \times 3, 9 = 3 \times 3, \ldots, 3000 = 1000 \times 3$$

   there are 1000 multiples of 3 from 3 to 3000, inclusive. Similarly, there are 1000 multiples of 3 from -3000 to -3. Since 0 is also divisible by 3, there are $1000 + 1000 + 1$ or 2001 integers from -3000 to 3000, inclusive, that are divisible by 3.
3. **(2)** When a positive integer k is divided by 6, the remainder is 1, so let $k = 7$. Then $5k = 35$. When 35 is divided by 3, the remainder is 2.
4. **(53)** The problem is equivalent to asking, "How many integers from 1 to 100 are *not* divisible by either 3 or 5?" Make an organized list while being careful not to count any number divisible by both 3 and 5, twice.
   - There are 20 integers from 1 to 100 that are divisible by 5:
     5, 10, 15, 20, 25, . . . , 100.
   - There are 27 integers from 1 to 100 that are divisible by 3 but not by 5:
     3, 6, 9, 12, 8, . . . , 99.
   - Hence, $20 + 27 = 47$ integers from 1 to 100 are divisible by either 3 or 5. Of the 100 circuit boards, $100 - 47 = 53$ were *not* checked by either of the two inspectors.

Lesson 3-4

MULTIPLE-CHOICE

1. **(E)** Multiplying both sides of the given equation, $2b = -3$, by -2:

   $$-2(2b) = -2(-3)$$

   Since $-4b = 6$, then $1-4b = 1+ 6 = 7$.
2. **(C)** Using the given facts that a is a negative integer and b is a positive integer, determine whether each Roman numeral statement is true or false.

- I. $a + b$ may be negative, 0, or positive, depending on the particular values of a and b. For example, if $a = -2$ and $b = 2$, $a + b = 0$. If, however, $a = -3$, and $b = 4$, $a + b > 0$. Statement I is false.
- II. $\frac{b-a}{a} = \frac{b}{a} - \frac{a}{a} = \frac{b}{a} - 1$. Since a positive integer divided by a negative integer is negative, $\frac{b}{a}$ is negative, making $\frac{b}{a} - 1$ negative. Hence $\frac{b-a}{a} < 0$. Statement II is true.
- III. When b is an even integer, $a^b > 0$. Statement III, $a^b < 0$, is not always true.

Hence, only statement II must be true.

3. **(D)** The product of five negative numbers is negative. Four negative numbers yield

$$\underbrace{(-)\times(-)}_{(+)}\times\underbrace{(-)\times(-)}_{(+)}\times(+) = (+)$$

Thus, four numbers in the product, at most, could be negative.

4. **(A)** If k equals 0, 2, or -1, $k(k-2)(k+1)$ evaluates to 0. Hence, you can eliminate answer choices (B), (C), and (D). Plug each of the two remaining answer choices into $k(k-2)(k+1)$ to see which results in a negative value. For answer choice (A), $k = -2$:

$$\begin{aligned} k(k-2)(k+1) &= -2(-2-2)(-2+1) \\ &= -2(-4)(-1) \\ &= -8 \end{aligned}$$

The correct choice is (A).

5. **(C)** $p^2(2-5) + (-p)^2 = -3p^2 + p^2 = -2p^2$
6. **(C)** If $n + 5$ is an odd integer, then n could be -2 since $-2 + 5 = 3$.
7. **(D)** Determine whether each Roman numeral statement is true or false when $a < 0$ and $b > 0$.
   - I. $a + b$ may be negative, 0, or positive, depending on the particular values of a and b. For example, if $a = -2$ and $b = 2$, then $a + b = 0$. If $a = -3$ and $b = 2$, then $a + b < 0$. Hence, statement I is false.
   - II. Pick numbers for a and b. If $a = -3$ and $b = 2$, then $b - a = 2 - (-3) = 2 + 3 = 5$. Since $b - a > 0$, statement II is true.
   - III. Since the square of a negative number is positive, $a\left(\frac{a}{b}\right) = \frac{a^2}{b} > 0$, so statement III is true.

   Only Roman numeral statements II and III are true.
8. **(D)** Since $(-2)^3 = (-2)\times(-2)\times(-2) = -8$ and $(-3)^2 = (-3)\times(-3) = 9$, then $(-2)^3 + (-3)^2 = -8 + 9 = 1$

Questions 9 and 10.

From the diagram, $a < -1$, $-1 < b < 0$, $0 < c < 1$, and $d > 1$.

9. **(B)** Determine whether each Roman numeral statement is true or false.
   - I. Pick a number for c. If $c = \frac{1}{2}$, then $c^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Since $\frac{1}{4} < \frac{1}{2}$, $c^2 < c$. Hence, statement I is true.
   - II. Since $a < -1$, then $a^2 > 1$. For example, if $a = -3$, then $a^2 = 9$. Since $0 < c < 1$, $a^2 > c$. Hence, statement II is true.
   - III. Pick a number for b. If $b = -\frac{1}{2}$, then $\frac{1}{b} = -\frac{2}{1} = -2$. Since $-\frac{1}{2} > -2$, then $b = \frac{1}{b}$. Hence, statement III is false.

   Only Roman numeral statements I and II are true.
10. **(C)** Determine whether each Roman numeral statement is true or false.
    - I. On the basis of the diagram, pick numbers for a and d. If $a = -2$ and $d = 2$, then $ad = -4 < b$, so statement I is false.
    - II. Since ab is the product of two negative numbers, $ab > 0$. Since ad is the product of numbers with opposite signs, $ad < 0$. Since $ab > ad$, statement II is true.
    - III. Since $a < b$, the reciprocals of a and b have the opposite size relationship. Since $\frac{1}{a} > \frac{1}{b}$, statement III is true.

    Hence, only Roman numeral statements II and III are true.

11. **(B)** If $(y-3)^2 = 16$, then the number inside the parentheses must be either 4 or -4. If $y - 3 = 4$, then $y = 7$ and $y^2 = 49$. If $y - 3 = -4$, then $y = 3-4 = -1$, so $y^2 = 1$. The smallest possible value of y^2 is 1.
12. **(E)** It is given that $Y = 1 + 2 + 3 + \ldots + 10$ and $X = (-1) + (-2) + (-3) + \ldots + (-10) = -(1 + 2 + 3 \ldots + 10) = -Y$. Since $X = -Y$:
    - $X + Y = -Y + Y = 0$, which makes Roman numeral choice I false.
    - $Y - X = Y-(-Y) = Y + Y = 2Y$, so Roman numeral choice II must be true.
    - $(X)^2 = (-Y)^2$ or, equivalently, $X^2 = Y^2$, so Roman numeral choice III must be true.
    - Since only Roman numeral choices II and III must be true, the correct choice is (E).
13. **(A)** Rewrite a^2b^3c as $(a^2b^2)bc > 0$. Then determine whether each Roman numeral statement is true or false.
    - I. Since $(a^2b^2)bc > 0$, $a^2 > 0$, and $b^2 > 0$, it must be the case that $bc > 0$. Hence, statement I is true.
    - II. Using $(a^2b^2)bc > 0$, you cannot tell whether ac is positive or negative. Hence, statement II is false.
    - III. In the product $(a^2b^2)bc > 0$, there is no restriction on the signs of a and b, so their product can be positive or negative. Hence, statement III is false.

    Only Roman numeral statement I must be true.
14. **(A)** Since $a^2b^3c^5 = (a^2b^2c^4)bc < 0$ and $a^2b^2c^4$ is always positive, it must be the case that bc is negative.
15. **(B)** Determine whether each Roman numeral statement is true or false when $a \neq 0$.
    - I. $(-a)^2 = (-a) \times (-a) = a^2$ and $a^2 - 2a^2 = -a^2$. Since $(-a)^2 \neq -a^2$, statement I is false.
    - II. Since $-(b-a) = -b-(-a) - b + a = a - b$, statement II is true.
    - III. If $a > 0$, then $a > -a$. However, if $a < 0$, then $a < -a$, so statement III is false.

    Hence, only Roman numeral statement II is always true.

GRID-IN

1. **(10)** Since $p^2 = 16$, $p = 4$ or -4. Also, $q^2 = 36$, so $q = 6$ or -6. Hence, the largest possible value of $q - p$ is $6 - (-4) = 6 + 4 = 10$.
2. **(16)** Follow these steps:
    - Find the largest possible value of y. Since x^2 is always nonnegative, the largest possible value of $y = 1 - x^2$ occurs when x^2 has its smallest value. Since $-4 \leq x \leq 2$, the smallest value of x^2 is 0. The *largest* possible value of y is $y = 1 - x^2 = 1 - 0 = 1$.
    - Find the smallest possible value of y. The smallest possible value of $y = 1 - x^2$ occurs when x^2 has its largest value. The largest value of x^2 is $(-4)^2 = 16$. The *smallest* possible value of y is $y = 1 - x^2 = 1 - 16 = -15$.
    - Subtract. The number obtained when the *smallest* possible value of y is subtracted from the *largest* possible value of y is $1 - (-15) = 1 + 15 = 16$.