YII 数学　SAT 演習

●4-2 Equations with more than One Variable

1. If $6 = 2x + 4y$, what is the value of $x + 2y$?

(A) 2
(B) 3
(C) 6
(D) 8
(E) 12

2. If $\frac{a}{2} + \frac{b}{2} = 3$ what is the value of $2a + 2b$?

(A) 6
(B) 8
(C) 12
(D) 16
(E) 24

3. If $2s - 3t = 3t - s$, what is s in terms of t?

(A) $\frac{t}{2}$
(B) $2t$
(C) $t + 2$
(D) $\frac{t}{2} + 1$
(E) $3t$

4. If $a + b = 5$ and $\frac{c}{2} = 3$, what is the value of $2a + 2b + 2c$?

(A) 12
(B) 14
(C) 16
(D) 20
(E) 22

5. If $xy + z = y$, what is x in terms of y and z?

(A) $\frac{y + z}{y}$
(B) $\frac{y - z}{z}$
(C) $\frac{y - z}{y}$
(D) $1 - z$
(E) $\frac{z - y}{y}$

6. If $b(x + 2y) = 60$ and $by = 15$, what is the value of bx?

(A) 15
(B) 20
(C) 25
(D) 30
(E) 45

7. If $2x^2 + 3y^2 = 0$, what is the value of $3x + 2y$?

(A) -1
(B) 0
(C) 1
(D) 3
(E) 5

8. If $\frac{a - b}{b} = \frac{2}{3}$, what is the value of $\frac{a}{b}$?

(A) $\frac{1}{2}$
(B) $\frac{3}{5}$
(C) $\frac{3}{2}$
(D) $\frac{5}{3}$
(E) 2

9. If $s + 3s$ is 2 more than $t + 3t$, then $s - t =$

(A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) $\frac{3}{4}$
(E) 2

10. If $\frac{1}{p + q} = r$ and $p \neq -q$, what is p in terms of r and q?

(A) $\frac{rq - 1}{q}$
(B) $\frac{1 + rq}{q}$
(C) $\frac{r}{1 + rq}$
(D) $\frac{1 - rq}{r}$
(E) $\frac{1 - q}{rq}$

11. If $a = 2b = 5c$, then $4a$ is equal to which of the following expressions?

   I. $8c$
   II. $4b + 10c$
   III. $2b + 15c$

   (A) I, II, and III
   (B) II and III only
   (C) II only
   (D) III only
   (E) None

12. If $\frac{a+b+c}{3} = \frac{a+b}{2}$ then $c =$

   (A) $\frac{a-b}{2}$
   (B) $\frac{a+b}{2}$
   (C) $5a + 5b$
   (D) $\frac{a+b}{5}$
   (E) $-a - b$

13. If $wx = z$, which of the following expressions is equal to xz?

   (A) $\frac{w}{z^2}$
   (B) $\frac{w^2}{z}$
   (C) wz^2
   (D) w^2z
   (E) $\frac{z^2}{w}$

14. If the value of n nickels plus d dimes is c cents, what is n in terms of d and x?

   (A) $\frac{c}{5} - 2d$
   (B) $5c - 2d$
   (C) $\frac{c-d}{10}$
   (D) $\frac{cd}{10}$
   (E) $\frac{c+10d}{5}$

15. If $\frac{c}{d} - \frac{a}{b} = x$, $a = 2c$, and $b = 5d$, what is the value of $\frac{c}{d}$ in terms of x?

   (A) $\frac{2}{3}x$
   (B) $\frac{3}{4}x$
   (C) $\frac{4}{3}x$
   (D) $\frac{5}{3}x$
   (E) $\frac{7}{2}x$

16. If $kx - 4 = (k-1)x$, which of the following must be true?

   (A) $x = -5$
   (B) $x = -4$
   (C) $x = -3$
   (D) $x = 4$
   (E) $x = 5$

17. If $c = b + 1$ and $p = 4b + 5$, which of the following is an expression for p, in terms of c?

   (A) $4c$
   (B) $4c - 4$
   (C) $\frac{c+1}{4}$
   (D) $4c + 1$
   (E) $4c + 9$

18. If p and r are positive integers and $2p + r - 1 = 2r + p + 1$, which of the following must be true?

   I. p and r are consecutive integers.
   II. p is even.
   III. r is odd.

   (A) None
   (B) I only
   (C) II only
   (D) III only
   (E) I, II, and III

Grid-In

1. If $16 \times a^2 \times 64 = (4 \times b)^2$ and a and b are positive integers, then b is how many times greater than a?

2. If $3a - c = 5b$ and $3a + 3b - c = 40$, what is the value of b?

3. If $a = 2x + 3$ and $b = 4x - 7$, for what value of x is $3b = 5a$?

4. $$\frac{x}{8} + \frac{y}{5} = \frac{31}{40}$$

   In the equation above, if x and y are positive integers, what is the value of $x + y$?

●4-2 Equations with more than One Variable 解答・解説

1. **(B)** Dividing each member of the given equation, $6 = 2x + 4y$, by 2 will leave $x + 2y$ on the right side of the equation:
$$\frac{6}{2} = \frac{2x}{2} + \frac{4y}{2}$$
$$3 = x + 2y$$
The value of $x + 2y$ is 3.

2. **(C)** Multiplying each member of the given equation, $\frac{a}{2} + \frac{b}{2} = 3$, by 4 will make the left side of the equation equal to $2a + 2b$:
$$4\left(\frac{a}{2}\right) + 4\left(\frac{b}{2}\right) = 4(3)$$
$$2a + 2b = 12$$
The value of $2a + 2b$ is 12.

3. **(B)** For the given equation, $2s - 3t = 3t - s$, finding s in terms of t means solving the equation for s by treating t as a constant. Work toward isolating s by first adding $3t$ on each side of the equation:
$$2s = 3t + 3t - s$$
$$= 6t - s$$
Next, add s on each side of the equation:
$$2s + s = 6t$$
$$3s = 6t$$
$$s = \frac{6t}{3} = 2t$$

4. **(E)**
   - If $a + b = 5$, then $2(a + b) = 2(5)$, so $2a + 2b = 10$.
   - If $\frac{c}{2} = 3$, then $4\left(\frac{c}{2}\right) = 4(3)$, so $2c = 12$.

   Hence, $2a + 2b + 2c = 10 + 12 = 22$.

5. **(C)** If $xy + z = y$, then $xy = y - z$, so $x = \frac{y - z}{y}$.

6. **(D)** If $b(x + 2y) = 60$, then $bx + 2by = 60$. Since it is also given that $by = 15$:
$$bx + 2by = 60$$
$$bx + 2(15) = 60$$
$$bx + 30 = 60$$
$$bx = 60 - 30 = 30$$

7. **(B)** Since x^2 and y^2 are both greater than or equal to 0, the only values of x^2 and y^2 for which $2x^2 + 3y^2 = 0$ are $x^2 = y^2 = 0$. If $x^2 = y^2 = 0$, then $x = y = 0$, so
$$3x + 2y = 3(0) + 2(0) = 0$$

8. **(D)** If $\frac{a - b}{b} = \frac{2}{3}$, then
$$\frac{a}{b} - \frac{b}{b} = \frac{2}{3}$$
$$\frac{a}{b} - 1 = \frac{2}{3}$$
$$\frac{a}{b} = 1 + \frac{2}{3}$$
$$= \frac{5}{3}$$
The value of $\frac{a}{b}$ is $\frac{5}{3}$.

9. **(C)** Since it is given that $s + 3s$ is 2 more than $t + 3t$, $s + 3s = (t + 3t) + 2$ or, equivalently, $4s = 4t + 2$, so $4s - 4t = 2$. Dividing each member of the equation by 4 gives $\frac{4s}{4} - \frac{4t}{4} = \frac{2}{4}$, which simplifies to $s - t = \frac{1}{2}$.

10. **(D)** Since $\frac{1}{p + q} = r = \frac{r}{1}$, eliminate the fractions by cross-multiplying:
$$r(p + q) = 1(1)$$
$$rp + rq = 1$$
$$rp = 1 - rq$$
$$p = \frac{1 - rq}{r}$$

11. **(B)** Determine whether each Roman numeral expression is equal to $4a$.
   - I. Since $a = 2b = 5c$, then $a = 5c$, so
$$4a = 4(5c) = 20c$$

   Expression I is not equal to $4a$.
   - II. Rewrite $4a$ as $2a + 2a$. Since $a = 2b = 5c$, then $2a = 2(2b) = 4b$ and $2a = 2(5c) = 10c$, so
$$4a = 2a + 2a = 4b + 10c$$

   Expression II is equal to $4a$.
   - III. Rewrite $4a$ as $a + 3a$. Since $a = 2b$ and $a = 5c$, then $3a = 3(5c) = 15c$. Hence,
$$4a = a + 3a = 2b + 15c$$

   Expression III is equal to $4a$.

   Only Roman numeral expressions II and III are equal to $4a$.

12. **(B)** Since $\frac{a+b+c}{3} = \frac{a+b}{2}$, eliminate the fractions by cross-multiplying:

$$2(a+b+c) = 3(a+b)$$
$$2a+2b+2c = 3a+3b$$
$$2c = (3a-2a)+(3b-2b)$$
$$= a + b$$
$$c = \frac{a+b}{2}$$

13. **(E)** If $wx = z$, then $x = \frac{z}{w}$, so

$$xz = z\left(\frac{z}{w}\right) = \frac{z^2}{w}$$

14. **(A)** The value in cents of n nickels plus d dimes is $5n + 10d$, which you are told is equal to c cents. Hence, $5n + 10d = c$ or $5n = c - 10d$, so

$$n = \frac{c}{5} - \frac{10d}{5} = \frac{c}{5} - 2d$$

15. **(D)** Since $a = 2c$ and $b = 5d$, replace a in the equation $\frac{c}{d} - \frac{a}{b} = x$, with $2c$ and replace b with $5d$:

$$\frac{c}{d} - \frac{2c}{5d} = x$$
$$\frac{5c}{5d} - \frac{2c}{5d} = x$$
$$\frac{5c-2c}{5d} = x$$
$$\frac{3c}{5d} = x$$

To find the value of $\frac{c}{d}$ in terms of x, multiply both sides of the equation $\frac{3c}{5d} = x$ by the reciprocal of $\frac{3}{5}$:

$$\frac{5}{3}\left(\frac{3c}{5d}\right) = \frac{5}{3}x$$
$$\frac{c}{d} = \frac{5}{3}x$$

16. **(D)** If $kx - 4 = (k-1)x$, removing parentheses makes $kx - 4 = kx - x$ so $-4 = -x$ or $x = 4$

17. **(D)** If $c = b + 1$, then $b = c - 1$. Hence, $p = 4b + 5 = 4(c-1) + 5 = 4c - 4 + 5 = 4c + 1$.

18. **(A)** If $2p + r - 1 = 2r + p + 1$, then, after like terms are collected on the same side of the equation, $p = r + 2$ where p and r are given as positive integers.

- I. Since p is 2 more than r, p and r cannot be consecutive integers. Hence, Roman numeral choice I is false.
- II. Since $p = r + 2$, p can be either odd (if r is odd) or even (if r is even). Hence, Roman numeral choice II is false.
- III. Roman numeral choice III is also false since r can be either even or odd. Since none of the Roman numeral choices must be true, the correct choice is (A).

GRID-IN

1. **(8)** Since $(4 \times b)^2 = 4^2 \times b^2 = 16 \times b^2$,

$$16 \times a^2 \times 64 = 16 \times b^2$$
$$\not{16} \times a^2 \times 64 = \not{16} \times b^2$$
$$a^2 \times 64 = b^2$$
$$\sqrt{a^2 \times 64} = \sqrt{b^2}$$
$$a \times 8 = b$$

Hence, b is 8 times as great as a.

2. **(5)** Rearrange the terms of $3a + 3b - c = 40$ to get $(3a - c) + 3b = 40$. Because $3a - c = 5b$, $5b + 3b = 40$; so, $8b = 40$ and $b = \frac{40}{8} = 5$.

3. **(18)** If $a = 2x + 3$ and $b = 4x - 7$, when $3b = 5a$, x must satisfy the equation $3(4x - 7) = 5(2x + 3)$. Removing parentheses makes $12x - 21 = 10x + 15$. Collecting like terms gives $12x - 10x = 15 + 21$ or $2x = 36$ so $x = \frac{36}{2} = 18$.

4. **(5)** Multiplying both sides of $\frac{x}{8} = \frac{y}{5} = \frac{31}{40}$ by 40 produces the eqivalent equation $5x + 8y = 31$. Substitute consecutive positive integer values for x until you find one that makes y have a positive integer value.

x	$5x$	$y = \frac{31-5x}{8}$	
1	5	$y = \frac{31-5}{8} = \frac{26}{8}$	
2	10	$y = \frac{31-10}{8} = \frac{21}{8}$	
3	15	$y = \frac{31-15}{8} = \frac{16}{8} = 2$	Stop!

Hence, $x + y = 3 + 2 = 5$.