YII 数学　SAT 演習

●4-6 Systems of Equations

1. If $2x - 3y = 11$ and $3x + 15 = 0$, what is the value of y?

   (A) -7
   (B) -5
   (C) $\frac{1}{3}$
   (D) 3
   (E) 10

2. If $2a = 3b$ and $4a + b = 21$, then $b =$

   (A) 1
   (B) 3
   (C) 4
   (D) 7
   (E) 8

3. If $2p + q = 11$ and $p + 2q = 13$, then $p + q =$

   (A) 6
   (B) 8
   (C) 9
   (D) 12
   (E) 18

4. If $m + p + k = 70$, $p = 2m$, and $k = 2p$, then $m =$

   (A) 2
   (B) 5
   (C) 7
   (D) 10
   (E) 14

5. If $x - y = 3$ and $x + y = 5$, what is the value of y?

   (A) -4
   (B) -2
   (C) -1
   (D) 1
   (E) 2

6. If $5x + y = 19$ and $x - 3y = 7$, then $x + y =$

   (A) -4
   (B) -1
   (C) 3
   (D) 4
   (E) It cannot be determined from the information given.

7. If $x - 9 = 2y$ and $x + 3 = 5y$, what is the value of x?

   (A) -2
   (B) 4
   (C) 11
   (D) 15
   (E) 17

8. If $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$ and $\frac{1}{x} - \frac{1}{y} = \frac{3}{4}$, then $x =$

   (A) $\frac{1}{4}$
   (B) $\frac{1}{2}$
   (C) 1
   (D) 2
   (E) 4

9. If $5a + 3b = 35$ and $\frac{a}{b} = \frac{2}{5}$, what is the value of a?

   (A) $\frac{14}{5}$
   (B) $\frac{7}{2}$
   (C) 5
   (D) 7
   (E) 9

10. If $\frac{x}{y} = 6$, $\frac{y}{w} = 4$ and $x = 36$, what is the value of w?

    (A) $\frac{1}{2}$
    (B) $\frac{3}{2}$
    (C) 2
    (D) 4
    (E) 6

11. If $4r + 7s = 23$ and $r - 2s = 17$ then $3r + 3s =$

(A) 8
(B) 24
(C) 32
(D) 40
(E) 48

12. If $\frac{p-q}{2} = 3$ and $rp - rq = 12$, then $r =$

(A) -1
(B) 1
(C) 2
(D) 4
(E) It cannot be determined from the information given.

13. If $(a + b)^2 = 9$ and $(a - b)^2 = 49$, what is the value of $a^2 + b^2$?

(A) 17
(B) 20
(C) 29
(D) 58
(E) 116

14. If $\frac{r+s}{r} = 3$ and $\frac{t+r}{t} = 5$, what is the value of $\frac{s}{t}$?

(A) $\frac{1}{2}$
(B) $\frac{3}{5}$
(C) 4
(D) 8
(E) 16

15. If $3x + y = c$ and $x + y = b$, what is the value of x in terms of c and b?

(A) $\frac{c-b}{3}$
(B) $\frac{c-b}{2}$
(C) $\frac{b-c}{3}$
(D) $\frac{b-c}{2}$
(E) $\frac{c-b}{4}$

16. If $a + b = 11$ and $a - b = 7$, then $ab =$

(A) 6
(B) 8
(C) 10
(D) 12
(E) 18

$$x - z = 7$$
$$x + y = 3$$
$$z - y = 6$$

17. For the above system of three equations, $x =$

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

$$a = 4c$$
$$c = re$$
$$a = 5e$$

18. For the system of equations above, if $e \neq 0$, what is the value of r?

(A) $\frac{1}{20}$
(B) $\frac{4}{5}$
(C) $\frac{5}{4}$
(D) 1
(E) 20

19. If $r^8 = 5$ and $r^7 = \frac{3}{t}$, what is the value of r in terms of t?

(A) $\frac{5}{3}t$
(B) $\frac{3}{5}t$
(C) $5 - \frac{3}{t}$
(D) $3 + \frac{5}{t}$
(E) $\frac{t}{15}$

20. If $\frac{a}{b} = \frac{6}{7}$ and $\frac{a}{c} = \frac{2}{5}$, what is the value of $3b + c$ in terms of a?

(A) $12a$
(B) $9a$
(C) $8a$
(D) $6a$
(E) $4a$

Grid- In

1. If 5 sips + 4 gulps = 1 glass and 13 sips + 7 gulps = 2 glasses, how many sips equal a gulp?

2. If $2a = 9 - b$ and $4a = 3b - 12$, what is the value of a?

3. John and Sara each bought the same type of pen and notebook in the school bookstore, which does not charge sales tax. John paid \$5.55 for two pens and three notebooks, and Sara paid \$3.50 for one pen and two notebooks. How much does the school bookstore charge for one notebook?

●4-6 Systems of Equations 解答・解説

1. **(A)** First solve the equation that contains one variable. Since $3x + 15 = 0$, then $3x = -15$, so

$$x = \frac{-15}{3} = -5$$

Substituting -5 for x in the other equation, $2x - 3y = 11$, gives $2(-5) - 3y = 11$ or $-10 - 3y = 11$. Adding 10 to both sides of the equation makes $-3y = 21$, so

$$y = \frac{-21}{3} = -7$$

2. **(B)** If $2a = 3b$, then $4a = 6b$. Substituting $6b$ for $4a$ in $4a + b = 21$ gives $6b + b = 21$ or $7b = 21$, so

$$b = \frac{21}{7} = 3$$

3. **(B)** Add corresponding sides of the two given equations:

$$\begin{array}{r} 2p + q = 11 \\ +\ \ p + 2q = 13 \\ \hline 3p + 3q = 24 \end{array}$$

Dividing each member of $3p + 3q = 24$ by 3 gives $p + q = 8$.

4. **(D)** Since $p = 2m$ and $k = 2p$, then

$$k = 2(2m) = 4m$$

Substituting for p and k in $m + p + k = 70$ gives

$$\begin{aligned} m + 2m + 4m &= 70 \\ 7m &= 70 \\ m &= \frac{70}{7} = 10 \end{aligned}$$

5. **(D)** Eliminate y by adding corresponding sides of the two equations:

$$\begin{array}{l} \ x - y = 3 \\ +\,x + y = 5 \\ \hline 2x + 0 = 8, \end{array} \quad \text{so} \quad x = \frac{8}{2} = 4$$

Since $x = 4$ and $x - y = 3$, then $4 - y = 3$, so $y = 1$.

6. **(C)** Subtract corresponding sides of the two given equations:

$$\begin{array}{r} 5x + y = 19 \\ -(x - 3y = 7) \\ \hline \end{array} \quad \rightarrow + \begin{array}{r} 5x + y = 19 \\ -\,x + 3y = -7 \\ \hline 4x + 4y = 12 \end{array}$$

Dividing each member of the equation $4x + 4y = 12$ by 4 gives $x + y = 3$.

7. **(E)** Subtract corresponding sides of the two given equations:

$$\begin{array}{r} x + 3 = 5y \\ -(x - 9 = 2y) \\ \hline \end{array} \quad \rightarrow + \begin{array}{r} x + 3 = 5y \\ -x + 9 = -2y \\ \hline 0 + 12 = 3y \\ \frac{12}{3} = y \text{ or } y = 4 \end{array}$$

Since $y = 4$ and $x + 3 = 5y$, then $x + 3 = 5(4) = 20$, so

$$x = 20 - 3 = 17$$

8. **(D)** Eliminate y by adding corresponding sides of the given equations:

$$\begin{array}{r} \frac{1}{x} + \frac{1}{y} = \frac{1}{4} \\ +\frac{1}{x} + \frac{1}{y} = \frac{3}{4} \\ \hline \frac{2}{x} + 0 = \frac{4}{4} = 1 \end{array}$$

Since $\frac{2}{x} = 1$, then $x = 2$.

9. **(A)** In the equation $\frac{a}{b} = \frac{2}{5}$, cross-multiplying gives $5a = 2b$. Since $5a + 3b = 35$ and $5a = 2b$, then

$$\begin{aligned} 2b + 3b &= 35 \\ 5b &= 35 \\ b &= 7 \end{aligned}$$

Since $5a = 2b = 2(7) = 14$,

$$a = \frac{14}{5}$$

10. **(B)** If $\frac{x}{y} = 6$ and $x = 36$, then $\frac{36}{y} = 6$, so $y = 6$. Since $\frac{y}{w} = 4$ and $y = 6$,

$$\begin{aligned} \frac{6}{w} &= 4 \\ 4w &= 6 \\ w &= \frac{6}{4} = \frac{3}{2} \end{aligned}$$

11. (**B**) Add corresponding sides of the given equations:

$$\begin{array}{r} 4r + 7s = 23 \\ +\ \ r - 2s = 17 \\ \hline 5r + 5s = 40 \end{array}$$

Dividing each member of $5r + 5s = 40$ by 5 gives $r + s = 8$. Since $r + s = 8$, then

$$3r + 3s = 3(8) = 24$$

12. (**C**) If $\frac{p-q}{2} = 3$ and $rp - rq = 12$, then

$$p - q = 2(3) = 6$$

and

$$r(p - q) = 12$$

so $r(6) = 12$ or $6r = 12$. Hence,

$$r = \frac{12}{6} = 2$$

13. (**C**) If $(a + b)^2 = 9$, then

$$a^2 + 2ab + b^2 = 9$$

If $(a - b)^2 = 49$, then

$$a^2 - 2ab + b^2 = 49$$

Add corresponding sides of the two equations:

$$\begin{array}{r} a^2 + 2ab + b^2 = 9 \\ +\ a^2 - 2ab + b^2 = 49 \\ \hline 2a^2 + 0 \quad + 2b^2 = 58 \end{array}$$

Dividing each member of $2a^2 + 2b^2 = 58$ by 2 gives

$$a^2 + b^2 = 29$$

14. (**D**) Proceed as follows:

- Find the value of $\frac{s}{r}$. If $\frac{r+s}{r} = 3$, then

$$\frac{r}{r} + \frac{s}{r} = 3 \text{ or } 1 + \frac{s}{r} = 3$$

so $\frac{s}{r} = 2$.

- Find the value of $\frac{r}{t}$. If $\frac{t+r}{t} = 5$, then

$$\frac{t}{t} + \frac{r}{t} = 5 \text{ or } 1 + \frac{r}{t} = 5 \text{ so } \frac{r}{t} = 4.$$

- Multiply corresponding sides of the equations $\frac{s}{r} = 2$ and $\frac{r}{t} = 4$:

$$\frac{s}{r} \times \frac{r}{t} = 2 \times 4$$

$$\frac{s}{t} = 8$$

15. (**B**) Eliminate y by subtracting corresponding sides of the given equations:

$$\begin{array}{r} 3x + y = c \\ -\ \ x + y = b \\ \hline 2x + 0 = c - b \end{array}$$

so $x = \frac{c-b}{2}$.

16. (**E**) If $a + b = 11$ and $a - b = 7$, then adding corresponding sides of the two equations gives $2a = 18$, so

$$a = \frac{18}{2} = 9$$

If $a + b = 11$ and $a = 9$, then $9 + b = 11$, so

$$b = 11 - 9 = 2$$

Hence,

$$ab = (9)(2) = 18$$

17. (**D**) For the given system of three equations,

$$x - z = 7$$
$$x + y = 3$$
$$z - y = 6$$

add the equations two at a time to eliminate the variables y and z.

- Eliminate y by adding corresponding sides of the second and third equations. The result is $x + z = 9$.
- Eliminate z by adding $x + z = 9$ to the first equation. The result is $2x = 16$.

Hence, $x = \frac{16}{2} = 8$.

18. (**C**) If $a = 4c$, $c = re$, and $a = 5e$, then substituting the second equation into the first equation gives $a = 4re$. Since $a = 4re = 5e$ and $e \neq 0$, the coefficients of e must be equal, so $4r = 5$ and $r = \frac{5}{4}$.

19. (**A**) Divide corresponding sides of the given equations:

$$\frac{r^8}{r^7} = 5 \div \frac{3}{t}$$

$$r = 5 \times \frac{t}{3}$$

$$= \frac{5}{3}t$$

20. (**D**)

- If $\frac{a}{b} = \frac{6}{7}$, then $6b = 7a$, so

$$3b = \frac{7}{2}a$$

- If $\frac{a}{c} = \frac{2}{5}$, then $2c = 5a$, so

$$c = \frac{5}{2}a$$

- Hence,

$$3b + c = \frac{7}{2}a + \frac{5}{2}a = \frac{12}{2}a = 6a$$

GRID-IN

1. (**3**) Since

$$5 \text{ sips} + 4 \text{ gulps} = 1 \text{ glass}$$

and

$$13 \text{ sips} + 7 \text{ gulps} = 2 \text{ glasses}$$

then

$$\begin{aligned} 13 \text{ sips} + 7 \text{ gulps} &= 2\overbrace{(5 \text{ sips} + 4 \text{ gulps})}^{1 \text{ glass}} \\ &= 10 \text{ sips} + 8 \text{ gulps} \\ 13 - 10 \text{ sips} &= 8 - 7 \text{ gulps} \\ 3 \text{ sips} &= 1 \text{ gulp} \end{aligned}$$

2. (**1.5**) Write one equation underneath the other, aligning like terms in the same vertical column. Eliminate b by multiplying the first equation by 3 and then adding the result to the second equation:

$$\begin{array}{l} 2a = 9 - b \\ \underline{4a = 3b - 12} \end{array} \quad \overset{3\times}{\rightarrow} \quad \begin{array}{r} 6a = \quad 27 - 3b \\ \underline{+\ 4b = -12 + 3b} \\ 10a = 15 + 0 \\ a = \dfrac{15}{10} = 1.5 \end{array}$$

3. (**1.45**) If $p =$ the cost of a pen and $n =$ the cost of a notebook then

$$\begin{array}{l} 1p + 2n = 3.50 \\ \underline{2p + 3n = 5.55} \end{array} \quad \overset{2\times}{\rightarrow} \quad \begin{array}{r} 2p + 4n = 7.00 \\ \underline{-\ 2p + 3n = 5.55} \\ n = 1.45 \end{array}$$

The charge for one notebook is \$1.45.