YII 数学　SAT 演習

●4-7 Algebraic Inequalities

1. What is the largest integer value of p that satisfies the inequality $4 + 3p < p + 1$?

   (A) -2
   (B) -1
   (C) 0
   (D) 1
   (E) 2

2. If $-3 < 2x + 5 < 9$, which of the following CANNOT be a possible value of x?

   (A) -2
   (B) -1
   (C) 0
   (D) 1
   (E) 2

3. If the sum of a number and the original number increased by 5 is greater than 11, which could be a possible value of the number?

   (A) -5
   (B) -1
   (C) 1
   (D) 3
   (E) 4

4. If $0 < a^2 < b$, which of the following statements is (are) always true?

   I. $a < \frac{b}{a}$
   II. $a^4 < a^2 b$
   III. $\frac{a^2}{b} < 1$

   (A) I only
   (B) I and II
   (C) II and III
   (D) I and III
   (E) I, II, and III

5. What is the smallest integer value of x that satisfies the inequality $4 - 3x < 11$?

   (A) -3
   (B) -2
   (C) -1
   (D) 0
   (E) 1

6. If $a > b > c > 0$, which of the following statements must be true?

   I. $\frac{a-c}{b-a} > \frac{b-c}{b-a}$
   II. $ab > ac$
   III. $\frac{b}{a} > \frac{b}{c}$

   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) II and III

7. If $\frac{r}{3} < 15$ and $s = r + 4$, which of the following must be true?

   (A) $r < 5$
   (B) $r < 18$
   (C) $s < 9$
   (D) $s < 20$
   (E) $s < 49$

8. Which of the following statements must be true when $a^2 < b^2$ and a and b are not 0?

   I. $\frac{a^2}{a} < \frac{b^2}{a}$
   II. $\frac{1}{a^2} > \frac{1}{b^2}$
   III. $(a + b)(a - b) < 0$

   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) II and III

9. For how many integer values of b is $b + 3 > 0$ and $1 > 2b - 9$?

(A) Four
(B) Five
(C) Six
(D) Seven
(E) Eight

10. If $xy > 1$ and $z < 0$, which of the following statements must be true?

I. $x > z$
II. $xyz < -1$
III. $\frac{xy}{z} < \frac{1}{z}$

(A) I only
(B) II only
(C) III only
(D) II and III
(E) None

Grid-In

1. For what integer value of y is $y + 5 > 8$ and $2y - 3 < 7$?

2. If 2 times an integer x is increased by 5, the result is always greater than 16 and less than 29. What is the least value of x?

3. If $2 < 20x - 13 < 3$, what is one possible value for x?

4. $\frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \frac{1}{10} < \frac{1}{8} - \frac{1}{9} + \frac{1}{10} + \frac{1}{n}$

For the above inequality, what is the greatest possible positive integer value of n?

●4-7 Algebraic Inequalities 解答・解説

1. **(A)** *Solution 1*: Since $4 + 3p < p + 1$, then
$$3p - p < 1 - 4 \quad \text{or} \quad 2p < -3$$
so $p < -\frac{3}{2}$. Hence, the largest integer value for p is -2.
Solution 2: Plug each of the answer choices for p into $4 + 3p < p + 1$ until you find one that makes the inequality a true statement. Since choice (A) gives
$$4 + 3(-2) < (-2) + 1$$
there is no need to continue.

2. **(E)** *Solution 1*: Solve $-3 < 2x + 5 < 9$ by first subtracting 5 from each member. The result is $-8 < 2x < 4$. Now divide each member of this inequality by 2, obtaining $-4 < x < 2$. Examine each of the answer choices until you find one (E) that is not between -4 and 2. Since x is less than 2, 2 is not a possible value of x.
Solution 2: Plug each of the answer choices for x into $-3 < 2x + 5 < 9$ until you find one (E) that does not make the inequality a true statement.

3. **(E)** If the sum of a number, x, and the original number increased by 5, $x + 5$, is greater than 11, then $x + (x + 5) > 11$, so $2x + 5 > 11$. Then $2x > 6$, so $x > 3$. The only answer choice that is greater than 3 is (E).

4. **(C)** Determine whether each Roman numeral statement is always true when $0 < a^2 < b$.
   - I. From the given inequality, you know that $a^2 < b$. Although a^2 is positive, a may or may not be positive. If $a > 0$, then $a < \frac{b}{a}$. If $a < 0$, then dividing each side of $a^2 < b$ by a reverses the inequality sign, so $a > \frac{b}{a}$. Hence, statement I is not always true.
   - II. Multiplying both sides of $a^2 < b$ by a^2 gives $a^4 < a^2b$, so statement II is always true.
   - III. Since $b > 0$, dividing both sides of $a^2 < b$ by b gives $\frac{a^2}{b} < 1$, so statement III is always true.

   Only Roman numeral statements II and III are always true.

5. **(B)** *Solution 1*: If $4 - 3x < 11$, then $-3x < 7$, so $x > -\frac{7}{3}$. Since $-\frac{7}{3}$ is between -2 and -3, the smallest integer value of x that satisfies this inequality is -2.
Solution 2: Plug each of the answer choices for x, starting with (A), into $4 - 3x < 11$ until you find one that makes the inequality a true statement. Choice (A) gives
$$4 - 3(-3) < 11$$
$$13 < 11$$
which is not a true statement.
Choice (B) gives
$$4 - 3(-2) < 11$$
$$10 < 11$$
which is true, so there is no need to continue.

6. **(B)** Determine whether each Roman numeral statement is always true when $a > b > c > 0$.
   - I. Since $a > b$, then $a - c > b - c$, and $b - a$ represents a negative number. Hence, when both sides of the inequality $a - c > b - c$ are divided by $b - a$, a true inequality results only if the direction of the inequality is reversed. Thus,
   $$\frac{a-c}{b-a} < \frac{b-c}{b-a}$$
   so statement I is not always true.
   - II. Since $b > c$ and $a > 0$, multiplying both sides of the inequality $b > c$ by a results in the true inequality $ab > ac$, so statement II is always true.
   - III. Since $a > c$, then $\frac{1}{a} < \frac{1}{c}$. Since $b > 0$, multiplying both sides of the inequality $\frac{1}{a} < \frac{1}{c}$ by b produces the true inequality $\frac{b}{a} > \frac{b}{c}$, so statement III is not always true.

   Hence, only Roman numeral statement II is always true.

7. **(E)** Since $\frac{r}{3} < 15$, $\not{3}\left(\frac{r}{\not{3}}\right) < 3(15)$, so $r < 45$.
Since $s = r + 4$ and $r < 45$, $s < 45 + 4$ or $s < 49$.

8. **(E)** If $a^2 < b^2$ and a and b are not 0, then a and b may be either positive or negative numbers. Determine whether each Roman numeral statement must be true.

- I. If $a > 0$, then $\frac{a^2}{a} < \frac{b^2}{a}$. Since dividing both sides of an inequality by a negative number reverses the inequality sign, if $a < 0$, then $\frac{a^2}{a} < \frac{b^2}{a}$. Hence, statement I is not always true.
- II. Since $a^2 < b^2$, their reciprocals have the opposite size relationship, so $\frac{1}{a^2} > \frac{1}{b^2}$. Statement II is always true.
- III. Since $a^2 < b^2$, then $a^2 - b^2 < 0$. Factoring the left side of this inequality gives $(a + b)(a - b) < 0$, so statement III is always true.

Only Roman numeral statements II and III must be true.

9. **(D)** If $b + 3 > 0$, then $b > -3$. Since $1 > 2b - 9$, then $10 > 2b$, so $5 > b$ or $b < 5$. Since b is an integer, b may be equal to any of these seven integers: $-2, -1, 0, 1, 2, 3$, or 4.

10. **(C)** Determine whether each Roman numeral statement is always true when $xy > 1$ and $z < 0$.

- I. If $x > 0$, then $x > z$. However, the fact that x may be a negative number could mean that $x < z$, so statement I is not always true.
- II. Multiplying an inequality by a negative quantity ($z < 0$) reverses the direction of the inequality, so $(xy)z < (1)z$, or $xyz < z$. Since z may or may not be greater than or equal to -1, the inequality $xyz < -1$ may or may not be true. Hence, statement II is not always true.
- III. Dividing $xy > 1$ by a negative quantity reverses the direction of the inequality, so $\frac{xy}{z} < \frac{1}{z}$. Statement III is always true.

Only Roman numeral statement III is always true.

GRID-IN

1. **(4)** If $2y - 3 < 7$, then $2y < 10$, so $y < 5$. Since
$$y + 5 > 8 \quad \text{and} \quad 2y - 3 < 7$$
then $y > 3$ and at the same time $y < 5$. The integer for which the question asks must be 4.

2. **(6)** When 2 times an integer x is increased by 5, the result is always greater than 16 and less than 29, so $16 < 2x + 5 < 29$. Subtracting 5 from each member of this inequality gives $11 < 2x < 24$. Then
$$\frac{11}{2} < \frac{2x}{2} < \frac{24}{2}$$
so $5\frac{1}{2} < x < 12$. According to this inequality, x is greater than $5\frac{1}{2}$, so the least integer value of x is 6.

3. **(.76)** If $2 < 20x - 13 < 3$, adding 13 to each member of the combined inequality makes $15 < 20x < 16$ or $\frac{15}{20} < x < \frac{16}{20}$, which can also be written as $0.75 < x < 0.80$. Hence, one possible value for x is 0.76. Grid in as .76.

4. **(6)** Canceling identical terms on either side of the given inequality, $\frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \frac{1}{10} < \frac{1}{8} - \frac{1}{9} + \frac{1}{10} + \frac{1}{n}$, results in $\frac{1}{7} < \frac{1}{n}$ or, equivalently, $n < 7$. Hence, the greatest possible integer value for n is 6.