YII 数学　SAT 演習

## ●5-1 Translating from English to Algebra

1. Carl has 7 fewer than twice the number of course credits that Steve has. Steve has 5 more course credits than Gary. If Gary has 8 course credits, how many does Carl have?

   (A) 6
   (B) 19
   (C) 20
   (D) 27
   (E) 33

2. Which of the following expressions represents the phrase "3 less than 2 times $x$"?

   (A) $3 - 2x$
   (B) $2 - 3x$
   (C) $3x - 2$
   (D) $2x - 3$
   (E) $2(3 - x)$

3. When 4 times a number $n$ is increased by 9, the result is 21.

   Which of the following equations represents the statement above?

   (A) $4(n + 9) = 21$
   (B) $4n = 9 + 21$
   (C) $n + 4 \times 9 = 21$
   (D) $4n + 9 = 21$
   (E) $4 + 9n = 21$

4. If $x - 4$ is 2 greater than $y + 1$, then by how much is $x + 6$ greater than $y$?

   (A) 7
   (B) 8
   (C) 13
   (D) 14
   (E) 15

5. When 3 is subtracted from 5 times a number $n$, the result is 27.

   Which of the following equations represents the statement above?

   (A) $5n - 3 = 27$
   (B) $3n - 5 = 27$
   (C) $5(n - 3) = 27$
   (D) $3(n - 5) = 27$
   (E) $3 - 5n = 27$

6. If $\frac{1}{3}$ of a number is 4 less than $\frac{1}{2}$ of the number, the number is

   (A) 12
   (B) 18
   (C) 24
   (D) 30
   (E) 36

7. Susan weighs $p$ pounds. If Susan gains 17 pounds, she will weigh as much as Carol, who weighs 8 pounds less than Judy. If Judy weighs $x$ pounds, then Susan's weight, $p$, in terms of $x$ is

   (A) $x - 25$
   (B) $x - 9$
   (C) $x + 9$
   (D) $17x - 8$
   (E) $x + 25$

8. When 2 times a number $n$ is decreased by 5, the result is at least 11.

   Which of the following expressions represents the sentence above?

   (A) $2(n - 5) \geq 11$
   (B) $2n - 5 \leq 11$
   (C) $2n - 5 \geq 11$
   (D) $2n - 5 = 11$
   (E) $2(n - 5) \leq 11$

9. When 3 times a number $n$ is increased by 7, the result is at most 4 times the number decreased by 1.

   Which of the following expressions represents the sentence above?

   (A)  $3n + 7 \leq 4n - 1$
   (B)  $3n + 7 > 4n - 1$
   (C)  $3n + 7 \leq 4(n - 1)$
   (D)  $3n + 7 \geq 4(n - 1)$
   (E)  $3(n + 7) \leq 4(n - 1)$

10. If $t$ multiplied by the sum of $x$ and $2y$ is divided by $4y$, the result is

   (A)  $\dfrac{xt}{2} + \dfrac{yt}{4}$

   (B)  $\dfrac{xt}{4} + \dfrac{yt}{2}$

   (C)  $\dfrac{xt}{4y} + \dfrac{t}{2}$

   (D)  $\dfrac{xt}{2y} + \dfrac{t}{4y}$

   (E)  $\dfrac{xt + yt}{2y}$

11. If 4 less than $x$ is 1 more than $y$, what is $x$ in terms of $y$?

   (A)  $y - 3$
   (B)  $y + 1$
   (C)  $y + 3$
   (D)  $y + 4$
   (E)  $y + 5$

12. If the number $a - 5$ is 3 less than $b$, which expression has the same value as 1 more than $b$?

   (A)  $a - 2$
   (B)  $a - 1$
   (C)  $a$
   (D)  $a + 1$
   (E)  $a + 2$

13. If $x + y$ is 4 more than $x - y$, which of the following statements must be true?

   I.   $x = 2$
   II.  $y = 2$
   III. $xy$ can have more than one value.

   (A) I only
   (B) II only
   (C) III only
   (D) I and III only
   (E) II and III only

14. Jill's present weight is 14 pounds less than her weight a year ago. If her weight at that time was $\frac{9}{8}$ of her present weight, what is her present weight in pounds?

   (A) 98
   (B) 104
   (C) 112
   (D) 118
   (E) 120

15. A number $x$ is 3 less than 4 times the number $y$. Two times the sum of $x$ and $y$ is 9. Which of the following pairs of equations could be used to find the values of $x$ and $y$?

   (A)     $x = 4y - 3$
           $2(x + y) = 9$
   (B)     $y = 4x - 3$
           $2(x + y) = 9$
   (C)     $x = 4(y - 3)$
           $2(x + y) = 9$
   (D)     $y = 4(x - 3)$
           $2x + y = 9$
   (E)     $x = 4y - 3$
           $2(xy) = 9$

16. Arthur has 3 times as many marbles as Vladimir. If Arthur gives Vladimir 6 marbles, Arthur will be left with 4 more marbles than Vladimir. What is the total number of marbles that Arthur and Vladimir have?

(A) 36
(B) 32
(C) 30
(D) 24
(E) 21

17. A video store rents each DVD at the rate of $x$ dollars for the first day and $y$ dollars for each additional day the DVD is out. When Sara returns a DVD to this store, she is charged $c$ dollars. In terms of $x$, $y$, and $c$, what is the number of rental days for which Sara is charged?

(A) $x + cy$

(B) $\dfrac{c - x}{y}$

(C) $\dfrac{x + y}{c}$

(D) $x + y(c - 1)$

(E) $1 + \dfrac{c - x}{y}$

18. A computer program is designed so that, when a number is entered, the computer output is obtained by multiplying the number by 3 and then subtracting 4 from the product. If the output that results from entering a number $x$ is then entered, which expression represents, in terms of $x$, the final output?

(A) $3x - 8$
(B) $3x - 12$
(C) $9x - 8$
(D) $9x - 16$
(E) $6x + 9$

19. There are 36 identical cartons, $\frac{3}{4}$ of which must be taped. If every 3 cartons require 2 rolls of tape, what is the total number of rolls of tape that will be needed?

(A) 18
(B) 21
(C) 24
(D) 27
(E) 30

20. After a certain number of people enter an empty room, $\frac{2}{3}$ of the people who entered the room leave. After 2 more people leave, $\frac{1}{4}$ of the original number of people who entered the room remain. What was the original number of people who entered the empty room?

(A) 9
(B) 12
(C) 18
(D) 24
(E) 30

21. In a high school that has a total of 950 students, the number of seniors is $\frac{3}{4}$ of the number of juniors, and the number of juniors is $\frac{2}{3}$ of the number of sophomores. If this school has the same number of freshmen as sophomores, how many students are seniors?

(A) 120
(B) 150
(C) 180
(D) 200
(E) 300

## Grid-In

1. If $\frac{5}{4}$ of $x$ is 20, what number is $x$ decreased by 1?

2. Half the difference of two positive numbers is 10. If the smaller of the two numbers is 3, what is the sum of the two numbers?

3. In a certain college class, each student received a grade of A, B, C, or D or an "Incomplete." In this class, $\frac{1}{6}$ of the students received As, $\frac{1}{4}$ received Bs, $\frac{1}{3}$ received Cs, and $\frac{1}{6}$ received Ds. If 3 students received grades of "Incomplete," how many students were in the class?

1. **(B)** Since Steve has 5 more course credits than Gary and Gary has 8 course credits, Steve has $5 + 8$ or 13 course credits. Carl has 7 fewer than twice the number of course credits that Steve has, so Carl has $(2 \times 13) - 7$ or 19 course credits.

2. **(D)** The phrase "3 less than 2 times $x$" means $2x$ minus 3 or $2x - 3$.

3. **(D)** When 4 times a number $n$ $(= 4n)$ is increased by 9 $(= 4n + 9)$, the result is 21. Hence, the equation is $4n + 9 = 21$.

4. **(C)** If $x - 4$ is 2 greater than $y + 1$, then
$$x - 4 = (y + 1) + 2$$
$$= y + 3$$
$$x = y + 7$$
Since $x + 6 = (y + 7) + 6$ or $x + 6 = y + 13$, $x + 6$ is greater than $y$ by 13.

5. **(A)** When 3 is subtracted from 5 times a number $n$ $(= 5n - 3)$, the result is 27. Hence, the equation is $5n - 3 = 27$.

6. **(C)** *Solution 1*: Let $x$ represent the unknown number. Since $\frac{1}{3}$ of $x$ is 4 less than $\frac{1}{2}$ of $x$, $\frac{x}{3} = \frac{x}{2} - 4$. Multiplying each member of this equation by 6, the LCD of its denominators, gives $2x = 3x - 24$, so
$$-x = -24 \quad \text{or} \quad x = 24$$
*Solution 2*: Plug each of the answer choices into the statement of the problem until you find one (C) that works: $\frac{1}{3}$ of 24 $(= 8)$ is 4 less than $\frac{1}{2}$ of 24 $(= 12)$.

7. **(A)** Since Carol weighs 8 pounds less than Judy and Judy weighs $x$ pounds, Carol weighs $x - 8$ pounds. If Susan, who weighs $p$ pounds, gains 17 pounds, she will weigh $p + 17$ pounds, which is as much as Carol weighs. Hence, $p + 17 = x - 8$, so $p = x - 25$.

8. **(C)** The expression "at least" in the given sentence means greater than or equal to. If two times a number $n$ is decreased by 5 $(= 2n - 5)$, the result is at least $(\geq)$ 11. Hence, $2n - 5 \geq 11$.

9. **(A)** The expression "at most" in the given sentence means less than or equal to. When 3 times a number $n$ is increased by 7 $(= 3n + 7)$, the result is at most $(\leq)$ 4 times the number decreased by 1 $(= 4n - 1)$. Hence, $3n + 7 \leq 4n - 1$.

10. **(C)** The sum of $x$ and $2y$ is $x + 2y$, and $t$ multiplied by this sum is $t(x + 2y)$. When this expression is divided by $4y$, the result is
$$\frac{t(x + 2y)}{4y} = \frac{xt + 2yt}{4y}$$
$$= \frac{xt}{4y} + \frac{2\cancel{y}t}{\cancel{4y}_2}$$
$$= \frac{xt}{4y} + \frac{t}{2}$$

11. **(E)** If 4 subtracted from $x$ is 1 more than $y$, then $x - 4 = y + 1$, so $x = y + 1 + 4$ or $x = y + 5$.

12. **(B)** If the number $a - 5$ is 3 less than $b$, then
$$a - 5 = b - 3$$
$$a - 5 + 3 = b$$
$$a - 2 = b$$
Since 1 more than $b$ is $b + 1$, add 1 to each side of the equation $a - 2 = b$, obtaining $a - 2 + 1 = b + 1$ or $a - 1 = b + 1$. Hence, $a - 1$ has the same value as 1 more than $b$.

13. **(E)** If $x + y$ is 4 more than $x - y$, then $x + y = x - y + 4$ or $y + y = x - x + 4$, so $2y = 4$. Determine whether each Roman numeral statement is true or false.
   - I. The value of $x$ cannot be determined, so statement I is false.
   - II. Statement II is true since $2y = 4$, so $y = 2$.
   - III. Since the value of $x$ is not fixed, $xy$ can have more than one value for different values of $x$. Hence, statement III is true.
Only Roman numeral statements II and III are true.

14. **(C)** Jill's weight 1 year ago was $\frac{9}{8}$ of her present weight, so Jill lost $\frac{1}{8}$ of her weight. Since her present weight is 14 pounds less than her weight a year ago, 14 pounds represents $\frac{1}{8}$ of her present weight. Hence, Jill's present weight is $8 \times 14$ or 112 pounds.

15. **(A)** If the number $x$ is 3 less than 4 times the number $y$, then $x = 4y - 3$. If 2 times the sum of $x$ and $y$ is 9, then $2(x + y) = 9$. The two equations in choice (A) can be used to find the values of $x$ and $y$.

16. **(B)** Since Arthur has 3 times as many marbles as Vladimir, let $x$ represent the number of marbles Vladimir has, and let $3x$ represent the number of marbles Arthur has. If Arthur gives Vladimir 6 marbles, Arthur now has $3x - 6$ marbles and Vladimir has $x + 6$ marbles. Since Arthur is left with 4 more marbles than Vladimir,

$$3x - 6 = (x + 6) + 4$$
$$= x + 10$$
$$2x = 16$$
$$x = 8$$

Since $x = 8$, $3x = 24$. The total number of marbles that Arthur and Vladimir have is $x + 3x = 8 + 24$ or 32.

17. **(E)** Let $n$ represent the number of rental days. Since the video store charges $x$ dollars for the first day and $y$ dollars for each additional day that the DVD is out, for $n$ days the charge is $1x + (n - 1)y$. Since Sara is charged $c$ dollars, $c = x + (n - 1)y$. Now solve this equation for $n$ by multiplying each term inside the parentheses by $y$:

$$c = x + ny - y$$
$$y + c - x = ny$$
$$\frac{y}{y} + \frac{c}{y} - \frac{x}{y} = \frac{ny}{y}$$
$$1 + \frac{c - x}{y} = n$$

18. **(D)** If $x$ is the number entered, then multiplying $x$ by 3 gives $3x$ and subtracting 4 from that product gives $3x - 4$. Since $3x - 4$ is then entered, the final output is obtained by multiplying $3x - 4$ by 3 and then subtracting 4. Thus, the final output is $3(3x - 4) - 4$, which simplifies to $9x - 12 - 4$ or $9x - 16$.

19. **(A)** Since $\frac{3}{4}$ of $36 = \frac{3}{4} \times 36 = 3 \times 9 = 27$, 27 of the 36 identical cartons must be taped. If every 3 cartons require 2 rolls of tape, then $\frac{27}{3} \times 2 = 9 \times 2 = 18$ rolls of tape will be needed.

20. **(D)** If the original number of people in the room is represented by $x$, then, after $\frac{2}{3}$ of the $x$ people in the room leave, $\frac{1}{3}x$ people remain. After two more people leave, $\frac{1}{3}x - 2$ people remain. Since this number represents $\frac{1}{4}$ of the $x$ people who originally entered the room, $\frac{1}{3}x - 2 = \frac{1}{4}x$. Multiplying each member of this equation by 12 gives $4x - 24 = 3x$, so $4x - 3x = 24$ and $x = 24$.

21. **(B)** Let $x$ represent the number of sophomores. Then $\frac{2}{3}x$ represents the number of juniors, and $\frac{3}{4}(\frac{2}{3}x)$ or $\frac{1}{2}x$ represents the number of seniors. Since this school has the same number of freshmen as sophomores, the number of freshmen is $x$. The total number of students is 950 so

$$x + x + \frac{2}{3}x + \frac{1}{2}x = 950$$
$$2x + \frac{2}{3}x + \frac{1}{2}x = 950$$
$$6(2x) + 6\left(\frac{2}{3}x\right) + 6\left(\frac{1}{2}x\right) = 6(950)$$
$$12x + 4x + 3x = 5700$$
$$19x = 5700$$
$$x = \frac{5700}{19}$$
$$= 300$$

Since $\frac{1}{2}x = \frac{1}{2}(300) = 150$, 150 students are seniors.

GRID-IN

1. **(15)** If $\frac{5}{4}$ of $x$ is 20, then $\frac{5}{4}x = 20$, so

$$x = \frac{4}{5}(20) = 16$$

Hence, $x$ decreased by 1 is $x - 1$ or 15.

2. **(26)** If half the difference of two positive numbers is 10, then the difference of the two positive numbers is 20. If the smaller of the two numbers is 3, then the other positive number must be 23 since $23 - 3 = 20$. Hence, the sum of the two numbers is $3 + 23$ or 26.

3. **(36)** You are given that in a certain college class each student received a grade of A, B, C, D or an "Incomplete." Since $\frac{1}{6} + \frac{1}{4} + \frac{1}{3} + \frac{1}{6}$ or $\frac{11}{12}$ of the number of students in the class received letter grades, $\frac{1}{12}$ of the students in the class received grades of "Incomplete." You are told that 3 students received this grade. Since these 3 students represent $\frac{1}{12}$ of the class, there were $3 \times 12$ or 36 students in the class.