YII 数学　SAT 演習　冬期６数学

●Basic Algebra

1. If $x = -3$, what is the value of the expression $x^2 + 3x + 3$?

   (A) −21
   (B) −15
   (C) −6
   (D) 3
   (E) 21

2. Let $<<x>> = 2x - 1$ for all positive integers. If $<<x>> = 15$, then $x =$

   (A) 6
   (B) 7
   (C) 8
   (D) 15
   (E) 16

3. $5z^2 - 5z + 4 - z(3z - 4) =$

   (A) $2z^2 - z + 4$
   (B) $2z^2 - 9z + 4$
   (C) $5z^2 - 8z + 8$
   (D) $5z^2 - 8z$
   (E) $2z^2 - 5z$

4. If $a = 2$, $b = -1$, and $c = 1$, which of the following must be true?

   I. $a + b + c = 2$
   II. $2a + bc = 4$
   III. $4a - b + c = 8$

   (A) I only
   (B) III only
   (C) I and II only
   (D) I and III only
   (E) I, II, and III

5. If $y \neq z$, then $\dfrac{xy - zx}{z - y} =$

   (A) x
   (B) 1
   (C) 0
   (D) −1
   (E) $-x$

6. For any number w, let $\# w \#$ be defined by the equation $\# w \# = -[w^2(w - 1)]$. What is the value of $\# -1 \#$?

   (A) −2
   (B) −1
   (C) 0
   (D) 1
   (E) 2

7. If $3 + x = 8$, what is the value of $5x$?

8. If $2(x - 40) = 3(x - 30)$, what is the value of x?

9. If $a = -1$ and $b = -2$, then $2a^2 - 2ab + b^2 =$

10. If $\dfrac{3}{a} = \dfrac{5}{4}$, what is the value of a?

11. If $q \times 34 \times 36 \times 38 = 17 \times 18 \times 19$, then $q =$

12. Which of the following is equivalent to $3x^2 + 18x + 27$?

    (A) $3(x^2 + 6x + 3)$
    (B) $3(x + 3)(x + 6)$
    (C) $3(x + 3)(x + 3)$
    (D) $3x(x + 6 + 9)$
    (E) $3x^2 + x(18 + 27)$

13. $(a^2 + b)^2 - (a^2 - b)^2 =$

    (A) $-4a^2b$
    (B) 0
    (C) $(2ab)2$
    (D) $4a^2b$
    (E) b^2

14. If $z \neq 0$, $x = \dfrac{4}{z}$, and $yz = 8$, then $\dfrac{x}{y} =$

    (A) 0.5
    (B) 1
    (C) 2
    (D) 16
    (E) 32

15. If $abc \neq 0$, then $\frac{a^2bc + ab^2c + abc^2}{abc} =$

(A) $a + b + c$
(B) $a + b + cabc$
(C) $a^3b^3c^3$
(D) $3abc$
(E) $2abc$

16. If $x \blacklozenge y = (x - y)^2$ for all integers, which of the following must be true?

I. $x \blacklozenge y = y \blacklozenge x$
II. $x \blacklozenge y = x \blacklozenge (-y)$
III. $x \blacklozenge (-y) = (-x) \blacklozenge y$

(A) I only
(B) III only
(C) I and II
(D) I and III
(E) II and III

17. If $x = 4$, what is $3y(4 - 2x)$ in terms of y?

(A) $-12y$
(B) $-6y$
(C) $-8y - 2$
(D) $12y - 4$
(E) 12

18. If $1 = |x|$, then x is

(A) -1
(B) 0
(C) 1
(D) -1 or 1
(E) 0 or 1

19. If $3n - 1 = 8 + 2m$, what is the value of $5(3n - 2m)$?

(A) 2
(B) 5
(C) 11
(D) 45
(E) 90

20. If $|2x + 9| = 33$, $x =$

(A) -21
(B) -12
(C) -12 or -21
(D) 12 or -21
(E) 12

21. If $x = -|6|^2 - |-4 + 7| \times 7$, then $x =$

(A) -74
(B) -57
(C) -23
(D) 15
(E) 36

22. If the product of 4, 5, and q is equal to the product of 5, p, and 2, and $pq \neq 0$, what is the value of $\frac{p}{q}$?

23. If $x = 2$ and $y = 3$, then $\frac{xy}{\frac{1}{x} + \frac{1}{y}} =$

24. The expression $\frac{3}{x - 1} - 6$ will equal 0 when x equals which of the following?

(A) -3
(B) $-\frac{2}{3}$
(C) $\frac{1}{2}$
(D) $\frac{3}{2}$
(E) 3

25. If $x > 1$ and $\frac{a}{b} = 1 - \frac{1}{x}$, then $\frac{b}{a} =$

(A) x
(B) $x - 1$
(C) $\frac{x - 1}{x}$
(D) $\frac{x}{x - 1}$
(E) $\frac{1}{x}$

26. If the numerical values of $(m+n)^2$ and $(m-n)^2$ are equal, then which of the following must be true?

I. $m+n=0$

II. $m-n=0$

III. $mn=0$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

27. If $a>0$, $b>\frac{1}{4}$, and $a+2b=1$, what is one possible value for a?

28. If $|q+17|=|q-17|$, and q is a real number, what is the value of q?

29. If $0<x<1$ and $0<y<1$, which of the following CANNOT be true?

(A) $xy>0$
(B) $xy>1$
(C) $xy>\frac{1}{2}$
(D) $x-y>0$
(E) $x+y>0$

30.
$$\begin{array}{r} \text{B E E} \\ +\ \text{S E A} \\ \hline \text{I D E A} \end{array}$$

In the correctly worked addition problem above, A, B, D, E, I, and S each represent a different digit. What is the smallest possible value of D?

31. If $xy=8$ and $x^2+y^2=16$, then $(x+y)^2=$

●Basic Algebra 解答

1. D

You want to find the value of the expression when x is -3, so plug in -3 for each x:

$$x^2 + 3x + 3 = (-3)^2 + 3(-3) + 3 = 9 + (-9) + 3 = 3$$

2. C

To solve for x, replace $\ll x \gg$ with the equivalent expression $2x - 1$ and set it equal to 15:

$$\begin{aligned} \ll x \gg &= 15 \\ 2x - 1 &= 15 \\ 2x &= 16 \\ x &= 8 \end{aligned}$$

3. A

Before you can carry out any other operations, you have to remove the parentheses. That's what "P" stands for in PEMDAS, an acronym for the order of operations in a mathematical expression: **P**arentheses, **E**xponents, **M**ultiplication, **D**ivision, **A**ddition, **S**ubtraction (see the SAT Math in a Nutshell section). Here you can use the distributive law:

$$\begin{aligned} z(3z - 4) &= z \times 3z - z \times 4 \\ &= 3z^2 - 4z \end{aligned}$$

But there's more to do—you're *subtracting* this whole expression from $5z^2 - 5z + 4$. Since subtraction is the inverse operation of addition, you must change the signs of $3z^2 - 4z$.

$$\begin{aligned} &5z^2 - 5z + 4 - (3z^2 - 4z) \\ &\quad = 5z^2 - 5z + 4 - 3z^2 + 4z \end{aligned}$$

Finally, combining like terms gives us:

$$\begin{aligned} &5z^2 - 5z + 4 - 3z^2 + 4z \\ &\quad = (5z^2 - 3z^2) + (-5z + 4z) + 4 \\ &\quad = 2z^2 - z + 4 \end{aligned}$$

4. A

Substitute $a = 2$, $b = -1$, and $c = 1$ into the statements.

Statement I: $$\begin{aligned} a + b + c &= 2 + (-1) + 1 \\ &= 2 \end{aligned}$$

Statement I is true, so eliminate choice (B).

Statement II: $$\begin{aligned} 2a + bc &= 2(2) + (-1)(1) \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

Statement II is false, so eliminate choices (C) and (E).

Statement III: $$\begin{aligned} 4a - b + c &= 4(2) - (-1) + 1 \\ &= 8 + 1 + 1 \\ &= 10 \end{aligned}$$

Statement III is false and the correct answer is choice (A).

5. E

Whenever you are asked to simplify a fraction that involves binomials, your first thought should be: Factor! Since x is in both terms of the numerator, we can factor out x and get

$$xy - zx = x(y - z)$$

Performing this operation on the original fraction, you find that

$$\frac{xy - zx}{z - y} = \frac{x(y - z)}{z - y}$$

Rewriting $(z - y)$ as $-1(y - z)$, you get:

$$\frac{x(y - z)}{-1(y - z)}$$

Now cancel $y - z$ from the top and bottom:

$$\frac{x}{-1} = -x$$

Note: There is a reason that you are told that $y \neq z$; otherwise you could have zero in the denominator, and the expression could be undefined.

6. E

Plug (-1) into the expression $-[w^2(w - 1)]$ and simplify:

$$-[(-1)^2((-1) - 1)]$$

Negative 1, squared, equals positive 1, so this equals:

$$\begin{aligned} &-[1(-1 - 1)] \\ &= -[1(-2)] \\ &= -(-2) = 2 \end{aligned}$$

7. 25

Subtract 3 from both sides of the given equation to find $x = 8 - 3 = 5$. Note that the question asks for the value of $5x$, not x. $5x = 5(5) = 25$.

8. 10

If $2(x - 40) = 3(x - 30)$, then $2x - 80 = 3x - 90$. Add $90 - 2x$ to both sides of this equation to find $10 = x$.

9. 2

Plug in -1 for each a and -2 for each b:

$$\begin{aligned} 2a^2 - 2ab + b^2 &= 2(-1)^2 - 2(-1)(-2) + (-2)^2 \\ &= 2 - 4 + 4 \\ &= 2 \end{aligned}$$

10. 2.4 or 12/5

Cross multiply, and then divide both sides by 5:

$$\frac{3}{a} = \frac{5}{4}$$

$$3 \times 4 = 5a$$

$$12 = 5a$$

$$a = \frac{12}{5} \text{ or } 2.4$$

11. 1/8 or .125

Don't multiply anything out! With such a bizarre-looking expression, there's usually a shortcut. Notice that each of the numbers on the right side is a factor of a number on the left side. So divide each side of the equation by $34 \times 36 \times 38$ to isolate q:

$$q = \frac{17 \times 18 \times 19}{34 \times 36 \times 38}$$

$$q = \frac{17}{34} \times \frac{18}{36} \times \frac{19}{38}$$

$$q = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8} \text{ or } .125$$

12. C

First factor out the number (3) common to all terms:

$$3x^2 + 18x + 27 = 3(x^2 + 6x + 9)$$

This is not an answer choice, so you must factor the polynomial.

$x^2 + 6x + 9$ is of the form $a^2 + 2ab + b^2$, with $a = x$ and $b = 3$.

So, $x^2 + 6x + 9 = (x + 3)^2$ or $(x + 3)(x + 3)$.

Therefore, $3x^2 + 18x + 27 = 3(x + 3)(x + 3)$.

An alternative method would be to multiply out the answer choices, and see which matches $3x^2 + 18x + 27$.

Choice (A): $3(x^2 + 6x + 3) = 3x^2 + 18x + 9$. Reject.

Choice (B): $3(x + 3)(x + 6) = 3(x^2 + 6x + 3x + 18)$. Using FOIL:

$= 3(x^2 + 9x + 18)$
$= 3x^2 + 27x + 3(18)$. Reject.

Choice (C): $3(x + 3)(x + 3) = 3(x^2 + 3x + 3x + 9)$. Using FOIL:

$= 3(x^2 + 6x + 9)$
$= 3x^2 + 18x + 27$. Correct.

13. D

Multiply out each part of the expression using FOIL.

$$(a^2 + b)^2 = (a^2 + b)(a^2 + b)$$
$$= a^4 + a^2b + ba^2 + b^2$$
$$= a^4 + 2a^2 b + b^2$$
$$(a^2 - b)^2 = (a^2 - b)(a^2 - b)$$
$$= (a^2)^2 + a^2(-b) + (-b)a^2 + (-b)^2$$
$$= a^4 - 2a^2b + b^2$$

So, $(a^2 + b)^2 - (a^2 - b)^2$
$= (a^4 + 2a^2b + b^2) - (a^4 - 2a^2b + b^2)$
$= a^4 + 2a^2b + b^2 - a^4 + 2a^2b - b^2$
$= 2a^2b + 2a^2b$
$= 4a^2b$

14. A

Rearrange the first equation:

$x = \frac{4}{z}$.

Multiply both sides by z:

$xz = 4$.

So $xz = 4$ and $yz = 8$. That is:

$$\frac{xz}{yz} = \frac{4}{8}.$$

$\frac{x}{y} = \frac{4}{8} = \frac{1}{2}$, or 0.5.

15. A

The expression has three terms in the numerator, and a single term, abc, in the denominator. Since the three terms in the numerator each have abc as a factor, abc can be factored out from both numerator and denominator, and the expression can be reduced to a simpler form.

$$\frac{a^2bc + ab^2c + abc^2}{abc}$$

$$= \frac{a(abc) + b(abc) + c(abc)}{abc}$$

$$= \frac{(a + b + c)(abc)}{abc}$$

$$= a + b + c$$

16. D

Consider each statement separately. Statement I claims $(x - y)^2 = (y - x)^2$. For any two integers, $y - x$ is the same as $-(x - y)$. So the statement claims that the squares of two integers with the same absolute value, but different signs equal each other. This is true, and can be shown by picking numbers. Therefore statement I must be true.

Statement II claims $(x - y)^2 = [x - (-y)]^2$, which equals $(x + y)^2$. Clearly $x - y$ can have a totally different absolute value from $x + y$, so statement II doesn't have to be true. (You can pick numbers to show this.)

Statement III claims $[x - (-y)]^2 = (-x - y)^2$. That is $(x + y)^2 = [-(x + y)]^2$. The expression being squared in the right-hand side is the negative of the expression being squared on the left-hand side. As before, squaring two numbers with identical absolute values produces the same result. So statement III must be true. That makes choice (D) the right answer.

17. A

Plug $x = 4$ into the expression.

$3y(4 - 2x) = 3y(4 - 2[4]) = 3y(4 - 8) = 3y(-4) = -12y$

18. D

Every positive number is the absolute value of two numbers: itself and its negative. Remember that absolute value is always positive because it represents a distance: the distance of a number from zero on the number line. In this case, the numbers whose distance from zero on the number line is one, are 1 and -1.

19. D

You don't need to find n or m. First add 1 to both sides, giving you $3n = 9 + 2m$. Then subtract $2m$ from both sides and re-express the given equation as $3n - 2m = 9$. Then substitute 9 for $3n - 2m$ in the expression $5(3n - 2m)$ to find that $5(9) = 45$.

20. D

If $|2x + 9| = 33$, then $2x + 9 = 33$ or $2x + 9 = -33$, so $2x = 24$ or $2x = -42$, and $x = 12$ or $x = -21$.

21. B

In addition to keeping careful track of the absolute values in this question, remember to follow order of operations.

$$x = -|6|^2 - |-4 + 7| \times 7$$

$$x = -36 - |3| \times 7$$

$$x = -36 - 21$$

$$x = -57$$

22. 2

You're told that $4 \times 5 \times q = 5 \times p \times 2$. The number 5 is a common factor so you can cancel it from each side. You are left with $4q = 2p$ or $2q = p$. Dividing both sides by q in order to get the quotient $\frac{p}{q}$ on one side, you find $\frac{p}{q} = 2$.

23. 36/5 or 7.2

Plug in the given values:

$$\frac{xy}{\frac{1}{x} + \frac{1}{y}} = \frac{2 \times 3}{\frac{1}{2} + \frac{1}{3}}$$

$$= \frac{6}{\frac{3}{6} + \frac{2}{6}}$$

$$= \frac{6}{\frac{5}{6}}$$

$$= 6 \times \frac{6}{5}$$

$$= \frac{36}{5} \text{ or } 7.2$$

24. D

You are asked to find x when $\frac{3}{x - 1} - 6 = 0$. Clear the denominator by multiplying both sides by $x - 1$.

$$\frac{3}{x - 1}(x - 1) - 6(x - 1) = 0(x - 1)$$

$$3 - 6(x - 1) = 0$$

$$3 - 6x + 6 = 0$$

$$9 - 6x = 0$$

$$9 - 6x + 6x = 0 + 6x$$

$$9 = 6x$$

$$\frac{9}{6} = \frac{6x}{6}$$

$$\frac{3}{2} = x$$

So answer choice (D) is correct. You can check your answer by plugging $\frac{3}{2}$ into the original equation.

25. D

Since $\frac{b}{a}$ is the reciprocal of $\frac{a}{b}$, $\frac{b}{a}$ must be the reciprocal of $1 - \frac{1}{x}$ as well. Combine the terms in $1 - \frac{1}{x}$ and then find its reciprocal.

$$\frac{a}{b} = 1 - \frac{1}{x} = \frac{x}{x} - \frac{1}{x} = \frac{x - 1}{x}$$

Therefore, $\frac{b}{a} = \frac{x}{x - 1}$.

26. C

You want to know which of the options must be true, given that $(m + n)^2 = (m - n)^2$.

First, expand the expressions.

$$(m + n)^2 = m^2 + 2mn + n^2$$

$$(m - n)^2 = m^2 - 2mn + n^2$$

Since these two expressions are equal, write:

$$m^2 + 2mn + n^2 = m^2 - 2mn + n^2$$

You have m^2 and n^2 on each side of the equal sign. Subtract these from both sides to leave:

$$2mn = -2mn$$

$$mn = -mn$$

Zero is the only number for which this is true, so $mn = 0$.

This shows statement III is certainly correct so eliminate choices (A), (B), and (D), but what about choice (E) that states I, II, and III must be true?

The key word in this problem is "must." If $m = 0$ and $n = -6$, for example, then $mn = 0$ and the given squared expressions are equal. But $m + n = 0 + -6 = -6$, not zero. Likewise, $m - n = 0 - -6 = 6$, not zero. So I and II need not be true, and III is the only correct statement.

27. $0 < a < 1/2$

We know that b is greater than $\frac{1}{4}$ (and therefore positive) and that $a + 2b = 1$, so a and b must both be fractions between 0 and 1. We can narrow down a even more by noting that b is always more than $\frac{1}{4}$, so $2b$ must be more than $\frac{1}{2}$. Since $a + 2b = 1$ and $2b$ is more than $\frac{1}{2}$, a must be less than $\frac{1}{2}$. You can grid a fraction between 0 and $\frac{1}{2}$, such as $\frac{1}{3}$ or $\frac{3}{8}$, or you can grid a decimal such as 0.25 or 0.4.

28. 0

There are four possibilities to deal with:

Possibility 1:

$$q + 17 = q - 17$$

$$q + 34 = q$$

$$34 = 0$$

This is not possible, so there are no real values of q that fulfill this possibility.

Possibility 2:

$$-(q + 17) = q - 17$$

$$-q - 17 = q - 17$$

$$0 = 2q$$

$$0 = q$$

Possibility 3, $q + 17 = -(q - 17)$, functions similarly to Possibility 2.

Possibility 4, $-(q + 17) = -(q - 17)$, functions similarly to Possibility 1.

29. B

Since there are variables in the answer choices, picking numbers is a good strategy for solving this question. Try to pick numbers to prove that each answer choice can be true. For instance, $x = .8$ and $y = .7$ demonstrates that all answer choices except (B) can be true. Furthermore, (B) cannot be true, since x and y must both be positive fractions less than 1, so xy will always be less than 1.

30. 2

The first thing to notice about this problem is that $E + A$ equals either A or $10 + A$. Since each letter represents only one digit, $E + A$ must equal A, since E cannot be 10. Therefore, $E = 0$. If we plug this into the problem, we get:

$$\begin{array}{r} B\,0\,0 \\ +\,S\,0\,A \\ \hline I\,D\,0\,A \end{array}$$

so

$$\begin{array}{r} B \\ +\,S \\ \hline I\,D \end{array}$$

So I must be 1, and B plus S must be 12 or more (since if $B + S$ were 10 or 11, the second letter in the answer would be E or I, not D). Therefore, the smallest possible value of D is 2. Note that there are several different possibilities for A, B, and S, but we don't need to find these to solve the problem.

31. 32

First, multiply out $(x + y)^2$ using the FOIL method: $(x + y)^2 = x^2 + 2xy + y^2$.

Regroup the terms:

$$= (x^2 + y^2) + 2(xy)$$

Plug in the given values:

$$= 16 + 2(8) = 32.$$