YII 数学　SAT 演習 冬期 I

●Practice Test#I (No Calculator)

12

A line in the xy-plane passes through the origin and has a slope of $\frac{1}{7}$. Which of the following points lies on the line?

A) $(0,7)$
B) $(1,7)$
C) $(7,7)$
D) $(14,2)$

13

If $x > 3$, which of the following is equivalent to $\dfrac{1}{\frac{1}{x+2}+\frac{1}{x+3}}$?

A) $\dfrac{2x+5}{x^2+5x+6}$

B) $\dfrac{x^2+5x+6}{2x+5}$

C) $2x+5$

D) x^2+5x+6

14

If $3x - y = 12$, what is the value of $\dfrac{8^x}{2^y}$?

A) 2^{12}
B) 4^4
C) 8^2
D) The value cannot be determined from the information given.

15

If $(ax+2)(bx+7) = 15x^2 + cx + 14$ for all values of x, and $a + b = 8$, what are the two possible values for c ?

A) 3 and 5
B) 6 and 35
C) 10 and 21
D) 31 and 41

16

If $t > 0$ and $t^2 - 4 = 0$, what is the value of t ?

19

In a right triangle, one angle measures $x°$, where $\sin x° = \frac{4}{5}$. What is $\cos(90° - x°)$?

Questions 22 and 23 refer to the following information.

Annual Budgets for Different Programs in Kansas, 2007 to 2010

Program	Year			
	2007	2008	2009	2010
Agriculture/natural resources	373,904	358,708	485,807	488,106
Education	2,164,607	2,413,984	2,274,514	3,008,036
General government	14,347,325	12,554,845	10,392,107	14,716,155
Highways and transportation	1,468,482	1,665,636	1,539,480	1,773,893
Human resources	4,051,050	4,099,067	4,618,444	5,921,379
Public safety	263,463	398,326	355,935	464,233

The table above lists the annual budget, in thousands of dollars, for each of six different state programs in Kansas from 2007 to 2010.

22

Which of the following best approximates the average rate of change in the annual budget for agriculture/natural resources in Kansas from 2008 to 2010 ?

A) $50,000,000 per year
B) $65,000,000 per year
C) $75,000,000 per year
D) $130,000,000 per year

23

Of the following, which program's ratio of its 2007 budget to its 2010 budget is closest to the human resources program's ratio of its 2007 budget to its 2010 budget?

A) Agriculture/natural resources
B) Education
C) Highways and transportation
D) Public safety

24

Which of the following is an equation of a circle in the xy-plane with center $(0, 4)$ and a radius with endpoint $\left(\frac{4}{3}, 5\right)$?

A) $x^2 + (y-4)^2 = \frac{25}{9}$

B) $x^2 + (y+4)^2 = \frac{25}{9}$

C) $x^2 + (y-4)^2 = \frac{5}{3}$

D) $x^2 + (y+4)^2 = \frac{3}{5}$

25

$$h = -4.9t^2 + 25t$$

The equation above expresses the approximate height h, in meters, of a ball t seconds after it is launched vertically upward from the ground with an initial velocity of 25 meters per second. After approximately how many seconds will the ball hit the ground?

A) 3.5
B) 4.0
C) 4.5
D) 5.0

26

Katarina is a botanist studying the production of pears by two types of pear trees. She noticed that Type A trees produced 20 percent more pears than Type B trees did. Based on Katarina's observation, if the Type A trees produced 144 pears, how many pears did the Type B trees produce?

A) 115

B) 120

C) 124

D) 173

27

A square field measures 10 meters by 10 meters. Ten students each mark off a randomly selected region of the field; each region is square and has side lengths of 1 meter, and no two regions overlap. The students count the earthworms contained in the soil to a depth of 5 centimeters beneath the ground's surface in each region. The results are shown in the table below.

Region	Number of earthworms	Region	Number of earthworms
A	107	F	141
B	147	G	150
C	146	H	154
D	135	I	176
E	149	J	166

Which of the following is a reasonable approximation of the number of earthworms to a depth of 5 centimeters beneath the ground's surface in the entire field?

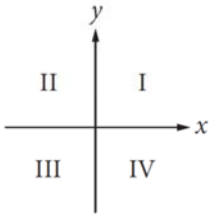
A) 150

B) 1,500

C) 15,000

D) 150,000

28



If the system of inequalities $y \geq 2x + 1$ and $y > \frac{1}{2}x - 1$ is graphed in the xy-plane above, which quadrant contains no solutions to the system?

A) Quadrant II

B) Quadrant III

C) Quadrant IV

D) There are solutions in all four quadrants.

29

For a polynomial $p(x)$, the value of $p(3)$ is -2. Which of the following must be true about $p(x)$?

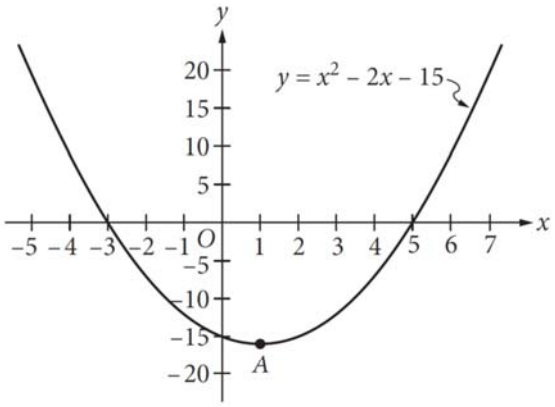
A) $x - 5$ is a factor of $p(x)$.

B) $x - 2$ is a factor of $p(x)$.

C) $x + 2$ is a factor of $p(x)$.

D) The remainder when $p(x)$ is divided by $x - 3$ is -2.

30



Which of the following is an equivalent form of the equation of the graph shown in the xy-plane above, from which the coordinates of vertex A can be identified as constants in the equation?

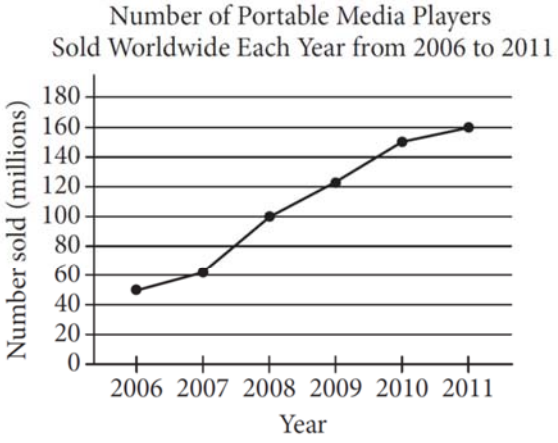
A) $y = (x + 3)(x - 5)$

B) $y = (x - 3)(x + 5)$

C) $y = x(x - 2) - 15$

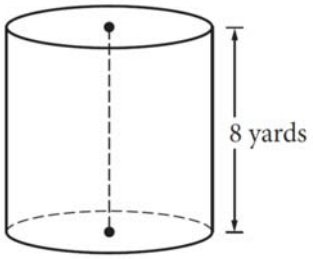
D) $y = (x - 1)^2 - 16$

33



According to the line graph above, the number of portable media players sold in 2008 is what fraction of the number sold in 2011 ?

35



A dairy farmer uses a storage silo that is in the shape of the right circular cylinder above. If the volume of the silo is 72π cubic yards, what is the diameter of the base of the cylinder, in yards?

Questions 37 and 38 refer to the following information.

Jessica opened a bank account that earns 2 percent interest compounded annually. Her initial deposit was \$100, and she uses the expression $\$100(x)^t$ to find the value of the account after t years.

37

What is the value of x in the expression?

38

Jessica's friend Tyshaun found an account that earns 2.5 percent interest compounded annually. Tyshaun made an initial deposit of \$100 into this account at the same time Jessica made a deposit of \$100 into her account. After 10 years, how much more money will Tyshaun's initial deposit have earned than Jessica's initial deposit? (Round your answer to the nearest cent and ignore the dollar sign when gridding your response.)

●Practice Test#1 (No Calculator) 解答

QUESTION 12

Choice D is correct. In the xy-plane, all lines that pass through the origin are of the form $y = mx$, where m is the slope of the line. Therefore, the equation of this line is $y = \frac{1}{7}x$, or $x = 7y$. A point with coordinates (a, b) will lie on the line if and only if $a = 7b$. Of the given choices, only choice D, (14, 2), satisfies this condition: $14 = 7(2)$.

Choice A is incorrect because the line determined by the origin (0, 0) and (0, 7) is the vertical line with equation $x = 0$; that is, the y-axis. The slope of the y-axis is undefined, not $\frac{1}{7}$. Therefore, the point (0, 7) does not lie on the line that passes the origin and has slope $\frac{1}{7}$.

Choices B and C are incorrect because neither of the ordered pairs has a y-coordinate that is $\frac{1}{7}$ the value of the corresponding x-coordinate.

QUESTION 13

Choice B is correct. To rewrite $\frac{1}{\frac{1}{x+2}+\frac{1}{x+3}}$, multiply by $\frac{(x+2)(x+3)}{(x+2)(x+3)}$. This results in the expression $\frac{(x+2)(x+3)}{(x+2)+(x+3)}$, which is equivalent to the expression in choice B.

Choices A, C, and D are incorrect and could be the result of common algebraic errors that arise while manipulating a complex fraction.

QUESTION 14

Choice A is correct. One approach is to express $\frac{8^x}{2^y}$ so that the numerator and denominator are expressed with the same base. Since 2 and 8 are both powers of 2, substituting 2^3 for 8 in the numerator of $\frac{8^x}{2^y}$ gives $\frac{(2^3)^x}{2^y}$, which can be rewritten as $\frac{2^{3x}}{2^y}$. Since the numerator and denominator of $\frac{2^{3x}}{2^y}$ have a common base, this expression can be rewritten as 2^{3x-y}. It is given that $3x - y = 12$, so one can substitute 12 for the exponent, $3x - y$, given that the expression $\frac{8^x}{2^y}$ is equal to 2^{12}.

Choice B is incorrect. The expression $\frac{8^x}{2^y}$ can be rewritten as $\frac{2^{3x}}{2^y}$, or 2^{3x-y}. If the value of 2^{3x-y} is 4^4, which can be rewritten as 2^8, then $2^{3x-y} = 2^8$, which results in $3x - y = 8$, not 12. Choice C is incorrect. If the value of $\frac{8^x}{2^y}$ is 8^2, then $2^{3x-y} = 8^2$, which results in $3x - y = 6$, not 12. Choice D is incorrect because the value of $\frac{8^x}{2^y}$ can be determined.

QUESTION 15

Choice D is correct. One can find the possible values of a and b in $(ax + 2)(bx + 7)$ by using the given equation $a + b = 8$ and finding another equation that relates the variables a and b. Since $(ax + 2)(bx + 7) = 15x^2 + cx + 14$, one can expand the left side of the equation to obtain $abx^2 + 7ax + 2bx + 14 = 15x^2 + cx + 14$. Since ab is the coefficient of x^2 on the left side of the equation and 15 is the coefficient of x^2 on the right side of the equation, it must be true that $ab = 15$. Since $a + b = 8$, it follows that $b = 8 - a$. Thus, $ab = 15$ can be rewritten as $a(8 - a) = 15$, which in turn can be rewritten as $a^2 - 8a + 15 = 0$. Factoring gives $(a - 3)(a - 5) = 0$. Thus, either $a = 3$ and $b = 5$, or $a = 5$ and $b = 3$. If $a = 3$ and $b = 5$, then $(ax + 2)(bx + 7) = (3x + 2)(5x + 7) = 15x^2 + 31x + 14$. Thus, one of the possible values of c is 31. If $a = 5$ and $b = 3$, then $(ax + 2)(bx + 7) = (5x + 2)(3x + 7) = 15x^2 + 41x + 14$. Thus, another possible value for c is 41. Therefore, the two possible values for c are 31 and 41.

Choice A is incorrect; the numbers 3 and 5 are possible values for a and b, but not possible values for c. Choice B is incorrect; if $a = 5$ and $b = 3$, then 6 and 35 are the coefficients of x when the expression $(5x + 2)(3x + 7)$ is expanded as $15x^2 + 35x + 6x + 14$. However, when the coefficients of x are 6 and 35, the value of c is 41 and not 6 and 35. Choice C is incorrect; if $a = 3$ and $b = 5$, then 10 and 21 are the coefficients of x when the expression $(3x + 2)(5x + 7)$ is expanded as $15x^2 + 21x + 10x + 14$. However, when the coefficients of x are 10 and 21, the value of c is 31 and not 10 and 21.

QUESTION 16

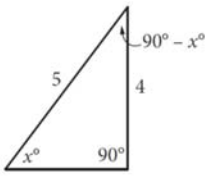
The correct answer is 2. To solve for t, factor the left side of $t^2 - 4 = 0$, giving $(t - 2)(t + 2) = 0$. Therefore, either $t - 2 = 0$ or $t + 2 = 0$. If $t - 2 = 0$, then $t = 2$, and if $t + 2 = 0$, then $t = -2$. Since it is given that $t > 0$, the value of t must be 2.

Another way to solve for t is to add 4 to both sides of $t^2 - 4 = 0$, giving $t^2 = 4$. Then, taking the square root of the left and the right side of the equation gives $t = \pm\sqrt{4} = \pm 2$. Since it is given that $t > 0$, the value of t must be 2.

QUESTION 19

The correct answer is $\frac{4}{5}$ or 0.8. By the complementary angle relationship for sine and cosine, $\sin(x°) = \cos(90° - x°)$. Therefore, $\cos(90° - x°) = \frac{4}{5}$. Either the fraction 4/5 or its decimal equivalent, 0.8, may be gridded as the correct answer.

Alternatively, one can construct a right triangle that has an angle of measure $x°$ such that $\sin(x°) = \frac{4}{5}$, as shown in the figure below, where $\sin(x°)$ is equal to the ratio of the length of the side opposite the angle measuring $x°$ to the length of the hypotenuse, or $\frac{4}{5}$.



Since two of the angles of the triangle are of measure $x°$ and 90°, the third angle must have the measure $180° - 90° - x° = 90° - x°$. From the figure, $\cos(90° - x°)$, which is equal to the ratio of the length of the side adjacent to the angle measuring $90° - x°$ to the hypotenuse, is also $\frac{4}{5}$.

●Practice Test#1（Calculator）解答

QUESTION 22

Choice B is correct. The amounts given in the table are in thousands of dollars. Therefore, the amount in the annual budget for agriculture/natural resources is actually \$488,106,000 in 2010 and \$358,708,000 in 2008. Therefore, the change in the budgeted amount is \$488,106,000 − \$358,708,000 = \$129,398,000. Hence, the average change in the annual budget for agriculture/natural resources from 2008 to 2010 is $\frac{\$129,398,000}{2}$ = \$64,699,000 per year. Of the options given, this average rate of change is closest to \$65,000,000 per year.

Choices A and C are incorrect and may result from errors in setting up or calculating the average rate of change. Choice D is incorrect; \$130,000,000 is the approximate total change in the annual budget for agriculture/natural resources from 2008 to 2010, not the average rate of change from 2008 to 2010.

QUESTION 23

Choice B is correct. The human resources budget in 2007 was 4,051,050 thousand dollars, and the human resources budget in 2010 was 5,921,379 thousand dollars. Therefore, the ratio of the 2007 budget to the 2010 budget is slightly greater than $\frac{4}{6} = \frac{2}{3}$. Similar estimates for agriculture/natural resources give a ratio of the 2007 budget to the 2010 budget of slightly greater than $\frac{3}{4}$; for education, a ratio of slightly greater than $\frac{2}{3}$; for highways and transportation, a ratio of slightly less than $\frac{5}{6}$; and for public safety, a ratio of slightly greater than $\frac{5}{9}$. Therefore, of the given choices, education's ratio of the 2007 budget to the 2010 budget is closest to that of human resources.

Choices A, C, and D are incorrect because the ratio of the 2007 budget to 2010 budget for each of the programs given in these choices is further from the corresponding ratio for human resources than the corresponding ratio for education.

QUESTION 24

Choice A is correct. The equation of a circle can be written as $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) are the coordinates of the center of the circle and r is the radius of the circle. Since the coordinates of the center of the circle are (0, 4), the equation of the circle is $x^2 + (y - 4)^2 = r^2$. The radius of the circle is the distance from the center, 0, 4), to the given endpoint of a radius, $\left(\frac{4}{3}, 5\right)$. By the distance formula, $r^2 = \left(\frac{4}{3} - 0\right)^2 + (5 - 4)^2 = \frac{25}{9}$. Therefore, an equation of the given circle is $x^2 + (y - 4)^2 = \frac{25}{9}$.

Choices B and D are incorrect. The equations given in these choices represent a circle with center (0, −4), not (0, 4). Choice C is incorrect; it results from using r instead of r^2 in the equation for the circle.

QUESTION 25

Choice D is correct. When the ball hits the ground, its height is 0 meters. Substituting 0 for h in $h = -4.9t^2 + 25t$ gives $0 = -4.9t^2 + 25t$, which can be rewritten as $0 = t(-4.9t + 25)$. Thus, the possible values of t are $t = 0$ and $t = \frac{25}{4.9} \approx 5.1$. The time $t = 0$ seconds corresponds to the time the ball is launched from the ground, and the time $t \approx 5.1$ seconds corresponds to the time after launch that the ball hits the ground. Of the given choices, 5.0 seconds is closest to 5.1 seconds, so the ball returns to the ground approximately 5.0 seconds after it is launched.

Choice A, B, and C are incorrect and could arise from conceptual or computation errors while solving $0 = -4.9t^2 + 25t$ for t.

QUESTION 26

Choice B is correct. Let x represent the number of pears produced by the Type B trees. Type A trees produce 20 percent more pears than Type B trees, or x, which can be represented as $x + 0.20x = 1.20x$ pears. Since Type A trees produce 144 pears, it follows that $1.20x = 144$. Thus $x = \frac{144}{1.20} = 120$. Therefore, the Type B trees produced 120 pears.

Choice A is incorrect because while 144 is reduced by approximately 20 percent, increasing 115 by 20 percent gives 138, not 144. Choice C is incorrect; it results from subtracting 20 from the number of pears produced by the Type A trees. Choice D is incorrect; it results from adding 20 percent of the number of pears produced by Type A trees to the number of pears produced by Type A trees.

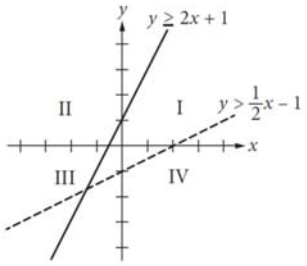
QUESTION 27

Choice C is correct. The area of the field is 100 square meters. Each 1-meter-by-1-meter square has an area of 1 square meter. Thus, on average, the earthworm counts to a depth of 5 centimeters for each of the regions investigated by the students should be about $\frac{1}{100}$ of the total number of earthworms to a depth of 5 centimeters in the entire field. Since the counts for the smaller regions are from 107 to 176, the estimate for the entire field should be between 10,700 and 17,600. Therefore, of the given choices, 15,000 is a reasonable estimate for the number of earthworms to a depth of 5 centimeters in the entire field.

Choice A is incorrect; 150 is the approximate number of earthworms in 1 square meter. Choice B is incorrect; it results from using 10 square meters as the area of the field. Choice D is incorrect; it results from using 1,000 square meters as the area of the field.

QUESTION 28

Choice C is correct. To determine which quadrant does not contain any solutions to the system of inequalities, graph the inequalities. Graph the inequality $y \geq 2x + 1$ by drawing a line through the y-intercept (0, 1) and the point (1, 3), as shown. The solutions to this inequality are all points contained on and above this line. Graph the inequality $y > \frac{1}{2}x - 1$ by drawing a dashed line through the y-intercept (0, −1) and the point (2, 0), as shown. The solutions to this inequality are all points above this dashed line.



The solution to the system of inequalities is the intersection of the regions above the graphs of both lines. It can be seen that the solutions only include points in quadrants I, II, and III and do not include any points in quadrant IV.

Choices A and B are incorrect because quadrants II and III contain solutions to the system of inequalities, as shown in the figure above. Choice D is incorrect because there are no solutions in quadrant IV.

QUESTION 29

Choice D is correct. If the polynomial $p(x)$ is divided by $x - 3$, the result can be written as $\frac{p(x)}{x - 3} = q(x) + \frac{r}{x - 3}$, where $q(x)$ is a polynomial and r is the remainder. Since $x - 3$ is a degree 1 polynomial, the remainder is a real number. Hence, $p(x)$ can be written as $p(x) = (x - 3)q(x) + r$, where r is a real number. It is given that $p(3) = -2$ so it must be true that $-2 = p(3) = (3 - 3)q(3) + r = (0)q(3) + r = r$. Therefore, the remainder when $p(x)$ is divided by $x - 3$ is −2.

Choice A is incorrect because $p(3) = -2$ does not imply that $p(5) = 0$. Choices B and C are incorrect because the remainder −2 or its opposite, 2, need not be a root of $p(x)$.

QUESTION 30

Choice D is correct. Any quadratic function q can be written in the form $q(x) = a(x - h)^2 + k$, where a, h, and k are constants and (h, k) is the vertex of the parabola when q is graphed in the coordinate plane. This form can be reached by completing the square in the expression that defines q. The equation of the graph is $y = x^2 - 2x - 15$.

Since the coefficient of x is -2, this equation can be written in terms of $(x - 1)^2 = x^2 - 2x + 1$ as follows: $y = x^2 - 2x - 15 = (x^2 - 2x + 1) - 16 = (x - 1)^2 - 16$. From this form of the equation, the coefficients of the vertex can be read as $(1, -16)$.

Choices A and C are incorrect because the coordinates of the vertex A do not appear as constants in these equations. Choice B is incorrect because it is not equivalent to the given equation.

QUESTION 33

The correct answer is $\frac{5}{8}$ or .625. Based on the line graph, the number of portable media players sold in 2008 was 100 million, and the number of portable media players sold in 2011 was 160 million. Therefore, the number of portable media players sold in 2008 is $\frac{100 \text{ million}}{160 \text{ million}}$ of the portable media players sold in 2011. This fraction reduces to $\frac{5}{8}$. Either 5/8 or its decimal equivalent, .625, may be gridded as the correct answer.

QUESTION 35

The correct answer is 6. The volume of a cylinder is $\pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. Since the storage silo is a cylinder with volume 72π cubic yards and height 8 yards, it follows that $72\pi = \pi r^2(8)$, where r is the radius of the base of the cylinder, in yards. Dividing both sides of the equation $72\pi = \pi r^2(8)$ by 8π gives $r^2 = 9$, and so the radius of the base of the cylinder is 3 yards. Therefore, the diameter of the base of the cylinder is 6 yards.

QUESTION 37

The correct answer is 1.02. The initial deposit earns 2 percent interest compounded annually. Thus at the end of 1 year, the new value of the account is the initial deposit of \$100 plus 2 percent of the initial deposit: $\$100 + \frac{2}{100}(\$100) = \$100(1.02)$. Since the interest is compounded annually, the value at the end of each succeeding year is the sum of the previous year's value plus 2 percent of the previous year's value. This is again equivalent to multiplying the previous year's value by 1.02. Thus, after 2 years, the value will be $\$100(1.02)(1.02) = \$100(1.02)^2$; after 3 years, the value will be $\$100(1.02)^3$; and after t years, the value will be $\$100(1.02)^t$. Therefore, in the formula for the value for Jessica's account after t years, $\$100(x)^t$, the value of x must be 1.02.

QUESTION 38

The correct answer is 6.11. Jessica made an initial deposit of \$100 into her account. The interest on her account is 2 percent compounded annually, so after 10 years, the value of her initial deposit has been multiplied 10 times by the factor $1 + 0.02 = 1.02$. Hence, after 10 years, Jessica's deposit is worth $\$100(1.02)^{10} = \$121.899$ to the nearest tenth of a cent. Tyshaun made an initial deposit of \$100 into his account. The interest on his account is 2.5 percent compounded annually, so after 10 years, the value of his initial deposit has been multiplied 10 times by the factor $1 + 0.025 = 1.025$. Hence, after 10 years, Tyshaun's deposit is worth $\$100(1.025)^{10} = \$128.008$ to the nearest tenth of a cent. Hence, Jessica's initial deposit earned \$21.899 and Tyshaun's initial deposit earned \$28.008. Therefore, to the nearest cent, Tyshaun's initial deposit earned \$6.11 more than Jessica's initial deposit.