2024 年 5 月 15 日　SAT　MATH　Exercises (Answers)

1

1 / 4 Let's expand the original polynomial to make it look like the second polynomial:

$$(y^2 - t^2)(y + k)$$

$$= (y^2 - t^2)y + (y^2 - t^2)k$$

2 / 4 Now we can distribute again.

$$= y^3 - t^2y + ky^2 - t^2k$$

$$= y^3 + ky^2 - t^2y - t^2k$$

Let's do a side by side comparison with the equivalent polynomial.

$$y^3 + 36y^2 - 9y + s$$

3 / 4 It follows that $k = 36$ and $-t^2 = -9$, and $-t^2k = s$. We solve for s:

$$\begin{aligned} s &= -t^2k \\ &= -9 \cdot 36 \\ &= -324 \end{aligned}$$

4 / 4 **The correct answer is:**

$$s = -324$$

2

1 / 4 We can rewrite the expression using the **square of sum** special factoring relationship.

$$a^2 + 2ab + b^2 = (a + b)^2$$

2 / 4 Notice that both $4x^2$ and 49 are perfect square terms:

- $4x^2 = (2x)^2$
- $49 = (7)^2$

3 / 4 If $a = 2x$ and $b = 7$, then the term $2ab$ would be equal to:

$$2(2x)(7) = 28x$$

Since $28x$ is a term in the expression, the expression satisfies the square of sum special factoring relationship.

For $a = 2x$ and $b = 7$:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$4x^2 + 28x + 49 = (2x + 7)^2$$

4 / 4 $(2x + 7)^2$ **is equivalent to the expression above.**

3

1 / 5 We can first find out what $4x + 2y$ is in terms of m and p, then factor the expression using the **square of sum** special factoring relationship.

$$a^2 + 2ab + b^2 = (a + b)^2$$

2 / 5 First, let's substitute $m^2 + p^2$ for x and $4mp$ for y into the expression $4x + 2y$:

$$4x + 2y = 4(m^2 + p^2) + 2(4mp)$$

$$= 4m^2 + 4p^2 + 8mp$$

$$= 4m^2 + 8mp + 4p^2$$

3 / 5 Notice that both $4m^2$ and $4p^2$ are perfect square terms:

- $4m^2 = (2m)^2$
- $4p^2 = (2p)^2$

4 / 5 If $a = 2m$ and $b = 2p$, then the term $2ab$ would be equal to:

$$2(2m)(2p) = 8mp$$

Since $8mp$ is a term in the expression, the expression satisfies the square of sum special factoring relationship.

For $a = 2m$ and $b = 2p$:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$4m^2 + 8mp + 4p^2 = (2m + 2p)^2$$

5 / 5 $(2m + 2p)^2$ **is equivalent to** $4x + 2y$.

4

1 / 5 Since all of the answer choices include the square of a binomial, let's complete the square for the given quadratic expression to see if there is a match.

2 / 5 Recall that to complete the square, we must group the terms with the x's together. Additionally, it is easiest to complete the square for a quadratic if the leading coefficient is 1.

3 / 5 This leads us to the following:

$$2x^2 + 4x + 3 = 2(x^2 + 2x) + 3$$

4 / 5 We can complete the square as follows:

$$= 2(x^2 + 2x + 1) + 3 - 2$$

$$= 2(x + 1)^2 + 1$$

Note that we really added $2 \cdot 1$ to the expression when completing the square. Therefore, in order to keep the expression balanced, we must subtract 2 from the expression.

5 / 5 **The correct answer is:**

$$2x^2 + 4x + 3 = 2(x + 1)^2 + 1$$

5

1 / 3 The volume V of a right square pyramid of vertical height h and base of side length s is:

$$V = \frac{s^2h}{3}$$

The volume of the original pyramid at Khufu is approximately:

$$V_1 = \frac{(230)^2(146.5)}{3}\ \text{m}^3$$

and the volume of the pyramid now is approximately:

$$V_2 = \frac{(230)^2(139)}{3}\ \text{m}^3$$

2 / 3 We can find the difference in the volumes using the distributive property:

$$\begin{aligned} V_1 - V_2 &= \frac{(230)^2(146.5)}{3} - \frac{(230)^2(139)}{3} \\ &= \frac{(230)^2(146.5 - 139)}{3} \\ &= \frac{(230)^2(7.5)}{3} \\ &= (230)^2(2.5) \\ &= 132{,}250\ \text{m}^3 \end{aligned}$$

3 / 3 **The volume of the pyramid decreased by approximately** $132{,}250\ \text{m}^3$.

6 1 / 4 Because the cones are identical and right, the height of each cone must be half the height of the top. Therefore, the height of each cone is 3 cm.

2 / 4 The formula for the volume of a cone uses the base radius and we are given the base diameter, 6 cm. The radius is half of the diameter, so the radius is 3 cm.

3 / 4 The volume of the top is the sum of the volumes of both cones. Because the cones are identical, we can double the volume of one cone:

$$V_{\text{top}} = 2\left(\frac{1}{3}\pi r^2 h\right)$$

$$= \frac{2}{3}\pi(3)^2(3)$$

$$= \frac{54}{3}\pi$$

$$= 18\pi\text{ cm}^3$$

4 / 4 **The correct answer is $18\pi\text{ cm}^3$.**

7 1 / 3 Since it takes him 72 breaths each containing 108π cubic centimeters of air to inflate the balloon, the balloon must have a volume of $72 \cdot 108\pi$ cubic centimeters.

2 / 3 Let r be the radius of the balloon in centimeters. Then we know that $\frac{4}{3}\pi r^3 = 72 \cdot 108\pi$, by the formula for the volume of a sphere.

So,

$$\frac{4}{3}\pi r^3 = 72 \cdot 108\pi$$

$$\frac{4}{3}r^3 = 9 \cdot 8 \cdot 54 \cdot 2$$

$$\frac{1}{3}r^3 = 9 \cdot 2 \cdot 54 \cdot 2$$

$$r^3 = 3 \cdot 9 \cdot 2 \cdot 54 \cdot 2$$

$$r^3 = 3 \cdot 9 \cdot 2 \cdot 27 \cdot 2 \cdot 2$$

$$r^3 = 3^3 \cdot 2^3 \cdot 27$$

$$r = 18$$

3 / 3 **So the radius of the balloon is 18 centimeters.**

8 1 / 4 The volume of a cylinder is the area of the base times the height.

2 / 4 The initial base of the pipe is a circle of radius 3 cm. So the area of the base is $\pi(3)^2 = 9\pi\text{ cm}^2$.

The initial volume of the pipe is $9\pi \cdot 11 = 99\pi\text{ cm}^3$.

3 / 4 The final base of the pipe is a circle of radius 4 cm. So the area of the base is $\pi(4)^2 = 16\pi\text{ cm}^2$.

The final volume of the pipe is $16\pi \cdot 11 = 176\pi\text{ cm}^3$.

$176\pi - 99\pi = 77\pi$, so the new pipe can hold $77\pi\text{ cm}^3$ more water than the old pipe.

According to the question, the new pipe can hold $w\pi\text{ cm}^3$ more water than the old pipe.

4 / 4 **The value of w is 77.**

9 1 / 5 We can plug $y = 28$ and $x = 12$ into the equation $y = kx$ to solve for k, then plug the value of k and $x = 9$ into the same equation to find y.

2 / 5 For $y = 28$ and $x = 12$, and value of k is:

$$y = kx$$

$$28 = k(12)$$

$$\frac{28}{12} = \frac{12k}{12}$$

$$\frac{\cancel{4} \cdot 7}{3 \cdot \cancel{4}} = k$$

$$\frac{7}{3} = k$$

3 / 5 Now, we can evaluate y using $k = \frac{7}{3}$ and $x = 9$:

$$y = kx$$

$$= \frac{7}{3}(9)$$

$$= 21$$

4 / 5 **Time-saver:** For two variables in a directly proportional relationship, if one variable is multiplied by a value, then the other variable must be multiplied by the same value. If you recognize that 9 is $\frac{3}{4}$ times 12, you can multiply 28 by $\frac{3}{4}$ and get 21 as well.

5 / 5 **21 is the value of y when $x = 9$.**

10 1 / 4 Let's set up a proportion to determine how many liters (L) of water Alexei uses for a bath. Let's use the variable b to represent the number of liters of water used in the bath.

$$\frac{\text{liter of water for shower}}{\text{time water runs for shower}} = \frac{\text{liters of water for bath}}{\text{time water runs for bath}}$$

$$\frac{62\text{ L}}{8\text{ minutes}} = \frac{b}{10.5\text{ minutes}}$$

2 / 4 Let's multiply by 10.5 minutes on both sides of the equation.

$$\frac{62\text{ L}}{8\text{ minutes}} \cdot 10.5\text{ minutes} = \frac{b}{10.5\text{ minutes}} \cdot 10.5\text{ minut}$$

$$81.375\text{ L} = b$$

Alexei uses 81.375 L of water when he takes a bath.

3 / 4 To determine how many more liters Alexei uses when he takes a bath than when he showers, we subtract.

$$\begin{array}{r} 81.375 \\ -62.000 \\ \hline 19.375 \end{array}$$

4 / 4 **To the nearest liter, Alexei uses 19 L more water when he takes a bath than when he takes a shower.**

11 **1 / 3** The ratio of dough to cream for the new "triple-stuffed" recipe is $\frac{1}{4}$ c dough to $\frac{3}{2}$ tbsp of cream.

2 / 3 To make a dozen triple-stuffed donuts, you need 12 times as much of the ingredients. Therefore, you need $12 \times \frac{1}{4} = 3$ c of dough and $12 \times \frac{3}{2} = 18$ tbsp of cream.

3 / 3 **So in order to make twelve triple-stuffed donuts, the ratio of dough to cream required is $3 : 18 = 1$ c : 6 tbsp.**

12 **1 / 4** Because the plastic shrank to a similar rectangle, we can set up a proportion to determine the width of the shrunken ornament. Let's use the variable w to represent the shrunken width of the drawing.

$$\frac{\text{original length of drawing}}{\text{original width of drawing}} = \frac{\text{shrunken length of drawi}}{\text{shrunken width of drawi}}$$

$$\frac{8\text{ cm}}{21\text{ cm}} = \frac{\left(2\frac{2}{3}\text{ cm}\right)}{w}$$

2 / 4 Let's multiply by the denominators of the fractions on both sides of the equation, then divide by the coefficient of w.

$$\frac{8\text{ cm}}{\cancel{21\text{ cm}}} \cdot w \cdot \cancel{21\text{ cm}} = \frac{(2\frac{2}{3}\text{ cm})}{\cancel{w}} \cdot \cancel{w} \cdot 21\text{ cm}$$

$$\frac{\cancel{8}\, w\ \cancel{\text{cm}}}{\cancel{8\text{ cm}}} = \frac{56\text{ cm} \cdot \cancel{\text{cm}}}{8\ \cancel{\text{cm}}}$$

$$w = 7\text{ cm}$$

3 / 4 To find the area of the shrunken ornament, we multiply the length by the width.

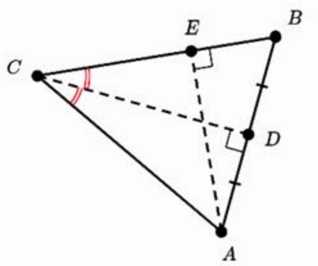
$$2\frac{2}{3} \cdot 7 = \frac{8}{3} \cdot 7$$
$$= \frac{56}{3}$$
$$= 18.\overline{6}$$

4 / 4 **The area of the shrunken ornament is $18.\overline{6}$ square centimeters.**

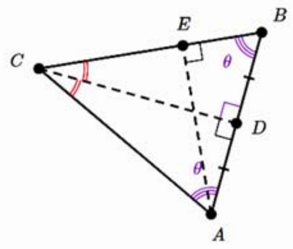
We can enter that answer as $\frac{56}{3}$, 18.66, or 18.67.

13 **1 / 5** Notice that line segment $\overline{CD}$ is a perpendicular bisector of the triangle.

2 / 5 Since $\overline{CD}$ is a perpendicular bisector, we know that $\angle DCA$ is congruent to $\angle DCB$.



3 / 5 Notice that $\angle BDC$ is a right angle. Since all the angles of a triangle must add up to $180°$, we know that the third angle of triangle ADC must be the same as the third angle in triangle BDC.



In other words, $\angle DAC$ is congruent to $\angle DBC$.

4 / 5 Since $\angle BAC$ is congruent to $\angle ABC$, triangle ABC has two congruent angles.

5 / 5 **Therefore, triangle ABC is an isosceles triangle.**

14 **1 / 5** First, notice that $8^{\frac{1}{2}}$ is common to both terms in the expression.

We can factor this out as shown below:

$$8^{\frac{5}{6}} - 8^{\frac{1}{2}} = 8^{\frac{5}{6}} - 8^{\frac{3}{6}}$$
$$= 8^{\frac{3}{6}} \cdot 8^{\frac{2}{6}} - 8^{\frac{3}{6}} \cdot 1$$
$$= 8^{\frac{1}{2}} \cdot 8^{\frac{1}{3}} - 8^{\frac{1}{2}} \cdot 1$$
$$= 8^{\frac{1}{2}}\left(8^{\frac{1}{3}} - 1\right)$$

2 / 5 Next, recall that $\sqrt[n]{x} = x^{\frac{1}{n}}$. Therefore, we know that $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$.

3 / 5 With this in mind, we can simplify the expression as follows:

$$8^{\frac{1}{2}}\left(8^{\frac{1}{3}} - 1\right) = 8^{\frac{1}{2}}(2 - 1)$$
$$= 8^{\frac{1}{2}} \cdot 1$$
$$= 8^{\frac{1}{2}}$$

4 / 5 Since $8^{\frac{5}{6}} - 8^{\frac{1}{2}} = 8^m$, it follows that $8^{\frac{1}{2}} = 8^m$ and that $m = \frac{1}{2}$.

5 / 5 $m = \frac{1}{2}$

15

1 / 4 Let's start by writing an equation to model the cost, p, of renting a moving truck at Mega Movers.

The following verbal model may be helpful:

$$[\text{total cost}] = [\text{cost per mile}] \cdot [\text{no. of miles}] + [\text{cost for tru}$$

Substituting in the given values gives us the following equation:

$$p = 0.95m + 19.50$$

2 / 4 Next, let's write an equation to model the cost, p, of renting a moving truck at U-Move-It.

Since we only pay a mileage fee at U-Move-It if $m > 20$, we can adjust the above verbal model as follows:

$$[\text{total cost}] = [\text{cost per mile}] \cdot [\text{no. of miles} > 20] + [\text{cost f}$$

Note that since m miles are driven, $m - 20$ represents the number of those miles that are beyond 20.

We can now fill in the verbal model as follows:

$$p = 0.45(m - 20) + 42$$

3 / 4 We must consider these equations simultaneously, and so we get the following system:

$$p = 0.95m + 19.5$$
$$p = 0.45(m - 20) + 42$$

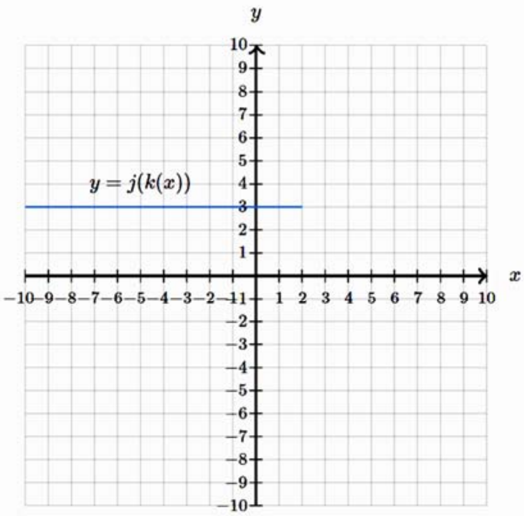
4 / 4 **The system that could be used to find the total mileage, m, that will make the cost, p, of renting a moving truck for one day equal at each rental company, assuming $m \geq 20$ is:**

$$p = 19.5 + 0.95m$$

$$p = 42 + 0.45(m - 20)$$

16

1 / 4 When $x \leq 2$, then $k(x)$ is equal to -3. Therefore, $j(k(x)) = j(-3)$ when $x \leq 2$. Next, from the graph of j, we see that $j(-3) = 3$. The graph of $y = j(k(x))$ must equal 3 when $x \leq 2$:



2 / 4 When $x \geq 2$, the graph of $y = k(x)$ has a slope of -1. In other words, as the value of x *increases* by 1, the value of $k(x)$ *decreases* by 1.

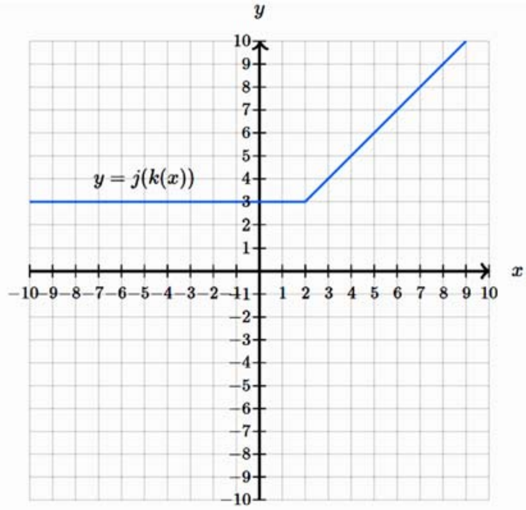
Because $k(x) \leq -3$ when $x \geq 2$, we look at $j(x)$ where $x \leq -3$ in order to find $j(k(x))$. In this region, $j(x)$ has a **slope** of -1. That is, as x *increases* by 1, $j(x)$ *decreases* by 1. ***This also***

2 / 4 When $x \geq 2$, the graph of $y = k(x)$ has a slope of -1. In other words, as the value of x *increases* by 1, the value of $k(x)$ *decreases* by 1.

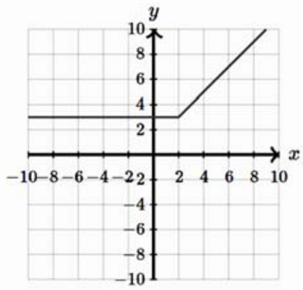
Because $k(x) \leq -3$ when $x \geq 2$, we look at $j(x)$ where $x \leq -3$ in order to find $j(k(x))$. In this region, $j(x)$ has a **slope** of -1. That is, as x *increases* by 1, $j(x)$ *decreases* by 1. ***This also ***means if x decreases by 1, then $j(x)$ increases by 1.***

Putting these two properties together, if we look at $j(k(x))$ then it must be that if $x \geq 2$ and x *increases* by 1, $k(x)$ *decreases* by 1, so $j(k(x))$ increases by 1. Therefore, $j(k(x))$ has a **slope** of 1 when $x \geq 2$.

3 / 4 Because $j(k(2)) = 3$ and $j(k(x))$ has a slope of 1 when $x \geq 2$, the rest of the graph is constructed by drawing the line of slope 1 going through the point $(2, 3)$ where $x \geq 2$:



4 / 4 **The correct graph is:**



17

1 / 2 Since the common denominator is $12p^3$, we will multiply the first fraction by $\frac{4}{4}$ and the second by $\frac{3p^2}{3p^2}$.

$$\frac{4}{4} \cdot \frac{3.5}{3p^3} + \frac{3p^2}{3p^2} \cdot \frac{7}{4p} = \frac{14}{12p^3} + \frac{21p^2}{12p^3}$$
$$= \frac{21p^2 + 14}{12p^3}$$

2 / 2 **The equivalent expression is:**

$$\frac{21p^2 + 14}{12p^3}$$

18

1 / 4 A *solution* to an equation is a value for the variable (in this case n) that makes the equation true.

2 / 4 Let's use the distributive property and combine like terms on the right-hand side:

$$8-3n=-3(n-1)+5$$

$$8-3n=-3n+3+5$$

$$8-3n=-3n+8$$

What values of n will make this equation a true statement?

3 / 4 The equation $8-3n=-3n+8$ will be true for *any* value of n, since the two sides of the equation are identical.

4 / 4 **The equation has infinitely many solutions.**

19

1 / 3 Let's look closely at this system of equations. If we rearrange the linear equation so that x is expressed in terms of y, then we can substitute into the circle equation and solve for y.

$$x+y=3$$

$$x=3-y$$

Now, let's substitute $3-y$ for x in the circle equation.

$$(3-y)^2+y^2=9$$

2 / 3 We can determine the y-coordinates in the solution set by setting this quadratic equation equal to zero and factoring (alternatively, we could use the quadratic formula or complete the square).

First, let's expand $(3-y)^2+y^2=9$.

$$(3-y)(3-y)+y^2=9$$

$$9-6y+y^2+y^2=9$$

$$9-9-6y+2y^2=0$$

$$2y^2-6y=0$$

$$2y(y-3)=0$$

Now that we have factored the quadratic equation, let's set each factor equal to zero and solve for the y-coordinates.

$$2y=0$$

$$y=0$$

and

$$y-3=0$$

$$y=3$$

3 / 3 **The product of the y-coordinates of the solutions to the system of equations is $0\cdot 3=0$.**

20

1 / 4 We can first factor out the constant 2 from the expression, then further factor the expression using the **difference of squares** special factoring relationship:

$$a^2-b^2=(a-b)(a+b)$$

2 / 4 Factoring out 2 from the expression gives us:

$$2x^2-50=2(x^2-25)$$

3 / 4 x^2 and 25 are both perfect squares. Since $25=5^2$, for $a=x$ and $b=5$:

$$a^2-b^2=(a-b)(a+b)$$

$$x^2-5^2=(x-5)(x+5)$$

x^2-25 is equivalent to $(x-5)(x+5)$, and $2(x^2-25)$ is equivalent to $2(x-5)(x+5)$.

4 / 4 **$2(x-5)(x+5)$ is equivalent to the expression above.**

21

1 / 3 In $y=mx+b$, the **slope-intercept form** of linear equations, m represents the slope and b represents the y-intercept of the line in the xy-plane.

- If y is greater than $mx+b$, the region above the line is shaded.
- If y is less than $mx+b$, the region below the line is shaded.

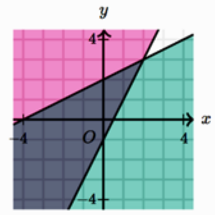
Notice that all four graphs have the same lines, but different shaded regions. As such, we only need to worry about which choice has the correctly shaded region based on the inequalities.

2 / 3 For $y\geq 2x-1$:

- The line defining the inequality has a slope of 2 and a y-intercept of -1.
- The region *above* the line should be shaded.

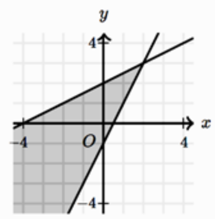
For $y\leq \frac{1}{2}x+2$:

- The line defining the inequality has a slope of $\frac{1}{2}$ and a y-intercept of 2.
- The region *below* the line should be shaded.



Since the lower left region is shaded by both inequalities, it represents the solution set to the system.

3 / 3 **The shaded region represents the solution set in the xy-plane to the system of inequalities above in the following:**



22

1 / 3 Let's look closely at the function:

$$f(x) = 0.07x + 2.34$$

In this context, x represents the number of years since 1950 and $f(x)$ represents the world population in billions of people.

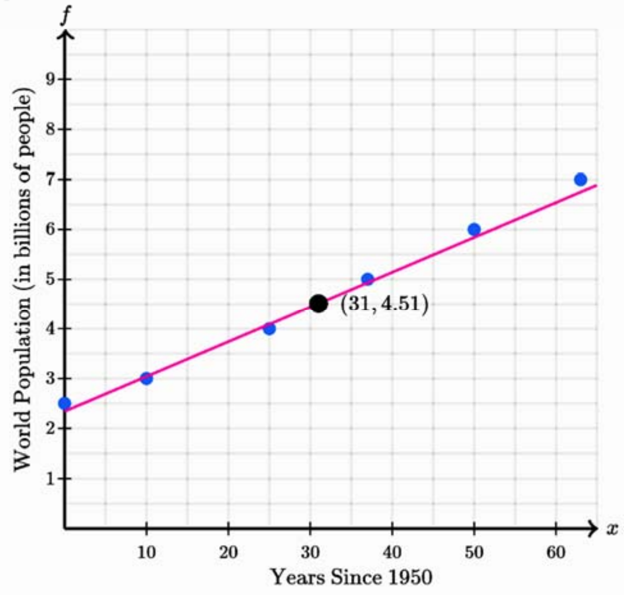
2 / 3 We want to use the model to estimate what the world population, $f(x)$, was in the year 1981.

Because $1981 - 1950 = 31$, let's substitute 31 for x and solve for $f(31)$:

$$\begin{aligned} f(31) &= 0.07(31) + 2.34 \\ &= 4.51 \end{aligned}$$

3 / 3 **According to the model, in the year 1981, the world population was approximately 4.51 billion people.**

World Population $(1950 - 2013)$



23

1 / 4 Add 36 to both sides so we can start isolating x on the left:

$$(x+1)^2 = 36$$

2 / 4 Take the square root of both sides to get rid of the exponent.

$$\sqrt{(x+1)^2} = \pm\sqrt{36}$$

3 / 4 Be sure to consider both positive and negative 6, since squaring either one results in 36.

$$\begin{aligned} x + 1 &= \pm 6 \\ x &= -1 \pm 6 \end{aligned}$$

Add and subtract 6 to find the two possible solutions:

$$x = 5 \quad \text{or} \quad x = -7$$

4 / 4 **The solutions are:**

$$x = 5\,, x = -7$$

24

1 / 5 The following verbal model may be helpful when thinking about this problem.

$$[\text{cost per foot}] \cdot [\text{number of feet}] = [\text{total cost of fencing}]$$

2 / 5 From the picture, we see that the total amount of fencing needed is $(2x + 30)$ ft. We also know that the fencing costs $\$7$ per foot. Finally, since Brett wants to make the fenced in area as wide as possible, he will be using his entire budget on fencing, and so the total cost of the fencing is $\$490$.

3 / 5 Substituting all of this into the above verbal model gives the following:

$$7(2x + 30) = 490$$

4 / 5 We can solve the equation for x as shown below.

$$7(2x + 30) = 490$$

$$2x + 30 = 70$$

$$2x = 40$$

$$x = 20$$

5 / 5 **The width of the fence is 20 ft.**

25

1 / 4 Scale models are proportional to the things they represent. Because this model is an enlargement, we can say:

$$36 : 1 = \text{model width} : \text{original width}$$

We make sure to align the larger number, 36, with the enlarged measurement.

2 / 4 Let's fill in the model width, 1170 millimeters, and rewrite the ratios using fractions to make it easier to manipulate. We can use the variable w to represent the width of the original scorpionfly.

$$\frac{36}{1} = \frac{1170}{w}$$

3 / 4 Let's solve this proportion by first multiplying by w, then dividing by the coefficient of w.

$$\frac{36}{1} \cdot w = \frac{1170}{\cancel{w}} \cdot \cancel{w}$$

$$\frac{\cancel{36}\,w}{\cancel{36}} = \frac{1170}{36}$$

$$w = 32.5$$

We recall that the units were in millimeters.

4 / 4 **The original scorpionfly had a width of 32.5 millimeters.**

26

1 / 3 The ratio between the sector's central angle θ and 2π radians is equal to the ratio between the sector's area, A_s, and the whole circle's area, A_c:

$$\frac{\theta}{2\pi} = \frac{A_s}{A_c}$$

2 / 3 Therefore, with $\theta = \frac{11}{6}\pi$, we have:

$$\frac{\frac{11}{6}\pi}{2\pi} = \frac{A_s}{36\pi}$$

$$\frac{11}{12} \times 36\pi = A_s$$

$$A_s = 33\pi$$

3 / 3 $A_s = 33\pi$