EXTRA PRACTICE: MULTIPLE-CHOICE ANSWER KEY

Math
Module (Harder)
1. C
2. A
3. D
4. D
5. A
6. A
7. D
8. B
9. C
10. D
11. $-\frac{9}{7}$
12. 175
13. C
14. 770
15. B
16. 1
17. C
18. B
19. B
20. 4
21. −9
22. C

EXTRA PRACTICE—MATH EXPLANATIONS

1. **C** The question asks for a value based on a percentage. Translate the English to math in bite-sized pieces. *Percent* means out of 100, so translate 64% as $\frac{64}{100}$. Since 64% of the trees in the random sample have ripe apples, apply that percent to the entire 250 trees in the orchard. Taking the percent *of* a number translates to multiplication, so 64% of 250 becomes $\frac{64}{100}(250)$. Use a calculator to get 160 trees with ripe apples. The correct answer is (C).

2. **A** The question asks for an expression that represents a specific situation. Translate the information into bite-sized pieces and eliminate after each piece. One piece of information says that the *carpenter spends 3 hours making each chair*, and another piece says that c is the number of chairs, so translate the total time spent making chairs as $3c$. Eliminate (B) and (D) because they do not include this term. Choice (C) includes $3c$ after expanding the left side of the equation using FOIL, but it also includes $7c$, which does not match the information in the question; eliminate (C). The correct answer is (A).

3. **D** The question asks for an equation that represents a graph. One method is to use the built-in graphing calculator. Enter each of the equations in the answer choices into the graphing calculator and see which line looks most like the line of best fit of the scatterplot given. Another method is to translate the information into bite-sized pieces and eliminate after each piece. The answer choices are all in slope-intercept form, $y = mx + b$, in which m is the slope and b is the y-intercept. The y-intercept is the point where $x = 0$, which is close to 1.5 on this graph. Eliminate (A) and (C) because they have negative y-intercepts. Compare the remaining answer choices. The difference between (B) and (D) is the sign of the slope. The line of best fit ascends from left to right, so it has a positive slope. Eliminate (B) because it has a negative slope. Using either method, the correct answer is (D).

4. **D** The question asks for a value on a graph. Find 5 on the x-axis: it is halfway between the labeled vertical line for 4 and the labeled vertical line for 6. Move up from there to the line of best fit, using the mouse pointer or scratch paper as a ruler if necessary. From there, move left to the y-axis to see that the value is between the unlabeled horizontal line for 7 and the labeled horizontal line for 8. Eliminate (A) and (B) because those values are not between 7 and 8. The find the exact y-coordinate of the point $(5, y)$, find the slope by using the formula $slope = \frac{y_2 - y_1}{x_2 - x_1}$. The graph has points at (0, 4) and (6, 8), so plug those values into the slope formula to get $slope = \frac{8 - 4}{6 - 0}$, which becomes $slope = \frac{4}{6}$, or $slope = \frac{2}{3}$. Now plug in the points at (0, 4) and $(5, y)$ to get $slope = \frac{y - 4}{5 - 0}$, which becomes $slope = \frac{y - 4}{5}$. The line has a slope of $\frac{2}{3}$, so $\frac{y - 4}{5} = \frac{2}{3}$. Cross-multiply to get $(5)(2) =$

$(y - 4)(3)$. Multiply and distribute to get $10 = 3y - 12$, then add 12 to both sides of the equation to get $22 = 3y$. Divide both sides of the equation by 3 to get $\frac{22}{3} = y$. The correct answer is (D).

5. **A** The question asks for the function that represents values given in a table. In function notation, the number inside the parentheses is the *x*-value that goes into the function, or the input, and the value that comes out of the function is the *y*-value, or the output. The table includes three input and output values, and the correct equation must work for every pair of values. Plug in values from the table and eliminate functions that don't work. Because 0 and 1 are likely to make more than one answer work, try the third row of the table and plug $x = 2$ and $h(x) = 37$ into the answer choices. Choice (A) becomes $37 = 2^2 + 7(2) + 19$, or $37 = 4 + 14 + 19$, and then $37 = 37$. This is true, so keep (A), but check the remaining answers just in case. Choice (B) becomes $37 = 2(2)^2 + 6(2) + 19$, or $37 = 8 + 12 + 19$, and then $37 = 39$. This is not true, so eliminate (B). Choice (C) becomes $37 = 3(2)^2 + 5(2) + 19$, or $37 = 12 + 10 + 19$, and then $37 = 41$; eliminate (C). Choice (D) becomes $37 = 6(2)^2 + 2(2) + 19$, or $37 = 24 + 4 + 19$, and then $37 = 47$; eliminate (D). The correct answer is (A).

6. **A** The question asks for a value given a function. In function notation, the number inside the parentheses is the *x*-value that goes into the function, or the input, and the value that comes out of the function is the *y*-value, or the output. The question provides an output value of 12, and the answers have numbers that could represent the *x*-value, *m*, so use the values in the answers. Start with one of the middle numbers and try (B), 9. Plug 9 into the function for *x* to get $g(9) = \frac{3}{4}(9) + 6$, which becomes $g(9) = \frac{27}{4} + 6$. This will not result in an integer, so it cannot equal the output value of 12; eliminate (B). The input value must be a multiple of 4 to result in an integer, so also eliminate (D). Try (C) next, and plug 12 into the function for *x* to get $g(12) = \frac{3}{4}(12) + 6$, which becomes $g(12) = 9 + 6$, or $g(12) = 15$. This is not 12, so eliminate (C). The correct answer is (A).

7. **D** The question asks for the value of an angle on a figure. Redraw the figure and labels on the scratch paper. When a line intersects two parallel lines, two kinds of angles are created: big and small. All of the small angles are equal to each other, all of the big angles are equal to each other, and any small angle plus any big angle = 180°. The angle marked $a°$ is a small angle, and the angle marked $b°$ is also a small angle. Thus, $a = b$, so set the expression equal to *a* and the expression equal to *b* equal to each other: $3x + 10 = 9x - 8$. Subtract $3x$ from both sides of the equation to get $10 = 6x - 8$, and then add 8 to both sides of the equation to get $18 = 6x$. Divide both sides of the equation by 6 to get $3 = x$. Plug $x = 3$ into the equation for angle *a* to get $a = 3(3) + 10$, which becomes $a = 9 + 10$, and then $a = 19$. Label both small angles, *a* and *b*, as 19. Since angle *c* is a large angle, its measure is $180 - 19 = 161$. The correct answer is (D).

8. **B** The question asks for the slope of a line that is parallel to another line. Parallel lines have the same slope, so calculate the slope of line l. The equation is in neither slope-intercept form nor standard form, so convert to one of those forms. To convert the equation of line l into slope-intercept form, $y = mx + b$, in which m is the slope, subtract $\frac{7}{2}$ from both sides of the equation to get $21y = -3x - \frac{7}{2}$. Divide both sides of the equation by 21 to get $y = -\frac{3}{21}x - \frac{7}{2(21)}$. Reduce the fractions to get $y = -\frac{1}{7}x - \frac{1}{6}$. The slope of line l is $-\frac{1}{7}$. Because line m is parallel to line l, its slope is also $-\frac{1}{7}$. The correct answer is (B).

9. **C** The question asks for the density of a geometric figure. Start by calculating the volume of the brick. Write down the formula for the volume of a rectangular prism, either from memory or after looking it up on the Reference Sheet. The formula is $V = lwh$. Plug in the information from the question to get $V = (24)(11.2)(7)$, which becomes $V = 1{,}881.6$ cubic centimeters. Next, use the units to figure out how to relate volume, mass, and density. Since density is in grams per cubic centimeter, mass is in grams, and volume is in cubic centimeters, density must be mass divided by volume. This happens to be the formula for density, which can be written as $D = \frac{m}{V}$. Plug in the values for mass and volume to get $D = \frac{3{,}100}{1{,}881.6}$, which becomes $D \approx 1.648$. Round to the nearest tenth to get $D = 1.6$. The correct answer is (C).

10. **D** The question asks for the value of a rate of decrease given a function that represents a specific situation. The value of the function is decreasing by a certain percent, so this question is about exponential decay. Write down the growth and decay formula, which is *final amount* = (*original amount*)(1 ± *rate*)$^{\text{number of changes}}$. The *number of changes* is given in terms of v, the number of vacant lots, and the question states that *the value of the house is predicted to decrease by r% for every 2 vacant lots*, so plug in 2 for v. The function becomes $R(2) = 120(0.98)^{\frac{1}{2}(2)}$, and then $R(2) = 120(0.98)^1$. Thus, the value changes once for every 2 vacant lots. The value of the house is decreasing, so the value in parentheses is (1 – *rate*). The question gives a value of 0.98, so 0.98 = 1 – *rate*. Add *rate* to both sides of the equation, and subtract 0.98 from both sides of the equation to get *rate* = 0.02. The question asks for the rate as a percentage, so multiply 0.02 by 100 to get 2%. The correct answer is (D).

11. $-\frac{9}{7}$ The question asks for a constant in a system of linear equations. When a linear equation has infinitely many solutions, the two sides of the equation are identical, and any value of x will make the equation true. Distribute on the right side of the equation to get $\frac{81}{23}x - \frac{27}{7} = 3ax + 3b$. The two terms with x must be equal, and the two terms without x must be equal. The question asks for the value of b, so focus on the terms without x. Set them equal to each other to get $-\frac{27}{7} = 3b$. Divide both sides of this equation by 3 to get $-\frac{27}{7(3)} = b$. Reduce the fraction to get $-\frac{9}{7} = b$. The correct answer is $-\frac{9}{7}$.

12. **175** The question asks for a value based on percentages and a specific situation. Translate the English to math in bite-sized pieces. Start with the sentence relating the number of seniors at school A and school B. Translate *is* as equals. *Percent* means out of 100, so translate 40% as $\frac{40}{100}$. Translate *fewer* as subtraction. Thus, 40% fewer is $1 - \frac{40}{100}$, or $\frac{60}{100}$. Use a variable for the number of seniors at school B, such as b. The equation becomes $168 = \frac{60}{100}b$. Multiply both sides of the equation by 100 and divide both sides of the equation by 60 to get $280 = b$. Next, translate the sentence relating the number of seniors at school B and school C. Translate *has* as equals, and translate 160% as $\frac{160}{100}$. Use the variable c to represent the number of seniors at school C. The equation becomes $280 = \frac{160}{100}c$. Multiply both sides of the equation by 100 and divide both sides of the equation by 160 to get $175 = c$. The correct answer is 175.

13. **C** The question asks for the value of a constant in a quadratic. To determine when a quadratic equation has exactly two real solutions, use the discriminant. The discriminant is the part of the quadratic formula under the square root sign and is written as $D = b^2 - 4ac$. When the discriminant is positive, the quadratic has exactly two real solutions; when the discriminant is 0, the quadratic has exactly one real solution; and when the discriminant is negative, the quadratic has no real solutions. Thus, the discriminant of this quadratic must equal a positive number. First, put the quadratic in standard form, which is $ax^2 + bx + c = 0$, by adding x^2 to both sides of the equation and subtracting $10x$ from both sides of the equation to get $x^2 - 10x + k = 0$. Now that the quadratic is in standard form, $a = 1$, $b = -10$, and $c = k$. Plug these into the discriminant formula to get $D = (-10)^2 - 4(1)(k)$, or $D = 100 - 4k$. Since there are exactly two real solutions, $100 - 4k > 0$. Add $4k$ to both sides of the inequality to get $100 > 4k$, then divide both sides of the inequality by 4 to get $25 > k$. The greatest integer that is less than 25 is 24. The correct answer is (C).

14. **770** The question asks for a mean, or average, based on two other means. For averages, use the formula $T = AN$, in which T is the *Total*, A is the *Average*, and N is the *Number of things*. For team X, the *number of things* is 4 and the *average* is 815, so the formula becomes $T = (815)(4)$, or $T = 3{,}260$. For team Y, the *number of things* is 6 and the *average* is 740, so the formula becomes $T = (740)(6)$, or $T = 4{,}440$. Add the *totals* for the two teams to get $3{,}260 + 4{,}440 = 7{,}700$; this is the *total* for team Z. Add the *numbers of things* for the two teams to get $4 + 6 = 10$; this is the *number of things* for team Z. The average formula for team Z becomes $7{,}700 = (A)(10)$. Divide both sides of the equation by 10 to get $770 = A$, which is the average, or mean, for team Z. The correct answer is 770.

15. **B** The question asks for an expression that must be an integer. Make the two sides of the equation look similar by using FOIL to expand the right side of the equation. The equation becomes $9x^2 - bx - 28 = fx^2 + fhx - gx - gh$. The two terms with x^2 are equivalent, so $9x^2 = fx^2$. Divide both sides of this equation by x^2 to get $9 = f$. Choice (A) is $\frac{28}{f}$, so plug in 9 for f to get $\frac{28}{9}$. This is not an integer, so eliminate (A). Next, work with the terms that do not have x^2 or x, and set -28 equal to $-gh$. Multiply both sides of this equation by -1 to get $28 = gh$. Choice (B) is $\frac{28}{g}$, so rewrite $28 = gh$ by dividing both sides of the equation by g to get $\frac{28}{g} = h$. The question states that h is an integer, so $\frac{28}{g}$ must also be an integer. The correct answer is (B).

16. **1** The question asks for the value of a constant in a quadratic. To determine when a quadratic equation has exactly one solution, use the discriminant. The discriminant is the part of the quadratic formula under the square root sign and is written as $D = b^2 - 4ac$. When the discriminant is positive, the quadratic has exactly two real solutions; when the discriminant is 0, the quadratic has exactly one real solution; and when the discriminant is negative, the quadratic has no real solutions. Thus, the discriminant of this quadratic must equal 0. First, substitute 3 for y in the second equation to get $3 = -2x^2 + 4x + k$, Next, put the quadratic in standard form, which is $ax^2 + bx + c = 0$, by subtracting 3 from both sides of the equation to get $0 = -2x^2 + 4x + k - 3$. Now that the quadratic is in standard form, $a = -2$, $b = 4$, and $c = k - 3$. Plug these into the discriminant formula to get $D = (4)^2 - 4(-2)(k - 3)$, or $D = 16 + 8(k - 3)$. Distribute the 8 to get $D = 16 + 8k - 24$, and then combine like terms to get $D = -8 + 8k$. Since there is exactly one real solution, $-8 + 8k = 0$. Add 8 to both sides of the equation to get $8k = 8$, and then divide both sides of the equation by 8 to get $k = 1$. The correct answer is 1.

17. **C** The question asks for the value of a constant in a system of linear equations. When two linear equations have infinitely many solutions, the two equations are the same line, and the equations are equivalent. Therefore, make the two equations look the same. First, put the equations in the same order by adding $\frac{6}{5}x$ to both sides of the first equation to get $\frac{6}{5}x + ay = 2$. Do the same with the second equation by subtracting x from both sides to get $5 = 3x + y$, and then switching the two sides of the equation to get $3x + y = 5$. The two equations now look like this:

$$\frac{6}{5}x + ay = 2$$
$$3x + y = 5$$

The two equations must equal each other, so make them both equal 10. Multiply the entire first equation by 5 to get $6x + 5ay = 10$. Multiply the entire second equation by 2 to get $6x + 2y = 10$. Set the y-terms equal to each other to get $5ay = 2y$, and then divide both sides of the equation by $5y$ to get $a = \frac{2}{5}$. The correct answer is (C).

18. **B** The question asks for the equation that defines a function that is graphed in the xy-plane. Since function h is related to function g, use knowledge of the transformation of graphs to tackle this question. When graphs are transformed, or translated, subtracting inside the parentheses shifts the graph to the right. Thus, $x - 3$ shifts the graph three units to the right. Find a point on the graph of $g(x)$, such as (5, 1). When the graph is shifted three units to the right, this point becomes (8, 1). Thus, the equation that defines function h must work for the point (8, 1). Plug $x = 8$ and $h(x) = 1$ into the answer choices, and eliminate any that don't work. Choice (A) becomes $1 = \frac{2(8-3)}{(8-3)}$, or $1 = 2$. This is not true, so eliminate (A). Choice (B) becomes $1 = \frac{2}{8-6}$, or $1 = \frac{2}{2}$, and then $1 = 1$. This is true, so keep (B), but check the remaining answers just in case. Choice (C) becomes $1 = \frac{2}{(8-3)}$, or $1 = \frac{2}{5}$; eliminate (C). Choice (D) becomes $1 = \frac{2}{8}$, or $1 = \frac{1}{4}$; eliminate (D). The correct answer is (B).

19. **B** The question asks for a value given information about the mean, or average, of a data set. For averages, use the formula $T = AN$, in which T is the *Total*, A is the *Average*, and N is the *Number of things*. Start by finding the total of the seven integers given in the question. There are 7 values, so the *number of things* is 7, and the *average* is given as 22. The average formula becomes $T = (22)(7)$, or $T = 154$. The question asks for the smallest integer greater than 10 that results in the full data set having an integer mean that is less than 22. The answers contain numbers that could be the value of the 8th integer, so use the values in the answers. The question states that all 8 integers are greater than 10, so eliminate (A) because 6 is less than 10. Try the next smallest answer and plug in 14 for the 8th integer. The *total* of the full data set becomes 154 + 14 = 168. The *number of things* is 8, so the average formula becomes $168 = (A)(8)$. Divide both sides of the equation by 8 to get 21. This is an integer less than 22, so it could be the mean of the full data set. Choice (B), 14, is the smallest integer that results in a mean that is an integer less than 22, so stop here. The correct answer is (B).

20. **4** The question asks for radius of a circle given an equation for its graph. The most efficient approach is to enter the equation into the built-in graphing calculator. Click on the gray dots at the maximum and minimum y-values to find the two ends of the diameter. The maximum y-value is at (1, 6), and the minimum y-value is at (1, –2). Since the two points have the same x-coordinate, the distance between them is the diameter of the circle. Find the difference to get a diameter of $6 - (-2) = 8$. The radius of a circle is half the diameter, so the radius is 4. The correct answer is 4.

21. **–9** The question asks for the least possible value of a constant in a quadratic. To determine when a quadratic equation has at least one real solution, use the discriminant. The discriminant is the part of the quadratic formula under the square root sign and is written as $D = b^2 - 4ac$. When the discriminant is positive, the quadratic has exactly two real solutions; when the discriminant is 0, the quadratic has exactly one real solution; and when the discriminant is negative, the quadratic has no real solutions. Thus, the discriminant of this quadratic must be equal to or greater than 0. First, distribute the x on the right side of the equation to get $4 = ax^2 + 12x$, then subtract 4 from both sides of the equation to get $0 = ax^2 + 12x - 4$. Now that the quadratic is in standard form, which is $ax^2 + bx + c = 0$, $a = a$, $b = 12$, and $c = -4$. Plug these into the discriminant formula to get $D = (12)^2 - 4(a)(-4)$, or $D = 144 + 16a$. Since there is at least one real solution, $144 + 16a \geq 0$. Subtract 144 from both sides of the inequality to get $16a \geq -144$. Divide both sides of the inequality by 16 to get $a \geq -9$. The least possible value of a is –9. The correct answer is –9.

22. **C** The question asks for a measurement on a geometric figure. Start by drawing two identical cylinders on the scratch paper, then label the figure with the given information. Label the height of each cylinder as 120. The answer choices contain numbers in increasing order, so use the values in the answers. Rewrite the answers on the scratch paper and label them "radius." Start with one of the answers in the middle and try (C), 6. Label the radius of each cylinder as 6, and then write down the formula for surface area. The surface area of a geometric figure is the sum of the areas of its faces. In the case of a cylinder, two faces are circles. The area of a circle is πr^2, so write this part of the surface area as $2(\pi r^2)$. The other part of the surface of a cylinder is the curved surface that connects the two circles. Think of this as the circumference of the circle at the base stacked as tall as the height, or circumference times height. The circumference of a circle is $2\pi r$, so write this part of the surface area as $2\pi rh$. Add these parts together to get the full formula for the surface area of a cylinder: $SA = 2\pi r^2 + 2\pi rh$.

Next, find the surface area of one of the cylinders using the given height of 120 and the plugged-in radius of 6. The formula becomes $SA = 2\pi(6)^2 + 2\pi(6)(120)$. Solve to get $SA = 72\pi + 1{,}440\pi$, and then $SA = 1{,}512\pi$. The question states that the surface area of one cylinder is A, so $A = 1{,}512\pi$. The question also states that the surface area of the combined cylinder is $\frac{41}{21}A$, so the surface area of the combined cylinder is $\frac{41}{21}(1{,}512\pi) = 2{,}952\pi$. Find the surface area of the combined cylinder and see if it matches this result. The radius is still 6, and the height becomes $120 + 120 = 240$. The surface area formula becomes $SA = 2\pi(6)^2 + 2\pi(6)(240)$. Solve to get $SA = 72\pi + 2{,}880\pi$, and then $SA = 2{,}952\pi$. This matches the information in the question, so stop here. The correct answer is (C).