

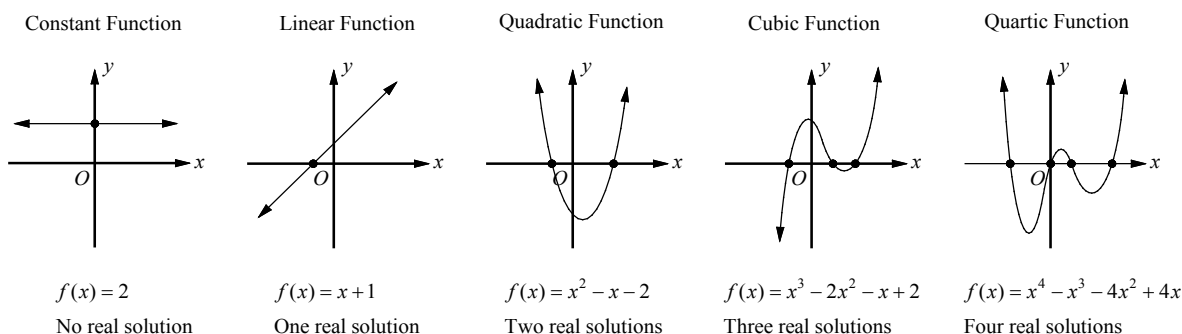
CHAPTER 13

Polynomial and Radical Functions

13-1. Polynomial Functions and Their Graphs

A **polynomial function** is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$, in which the coefficients $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ are real numbers and n is a nonnegative integer.

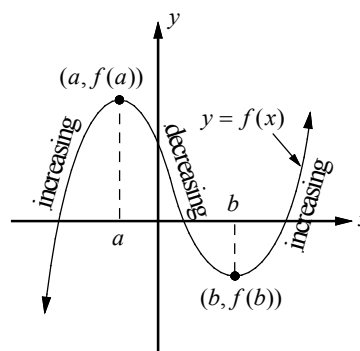
The degree of a polynomial function is its greatest exponent of x . The graphs of several polynomial functions are shown below. The maximum number of zeros, which are the x -intercepts, is equal to the degree of the function.



A function f is **increasing** on an interval if the value of f increases as x increases in the interval.

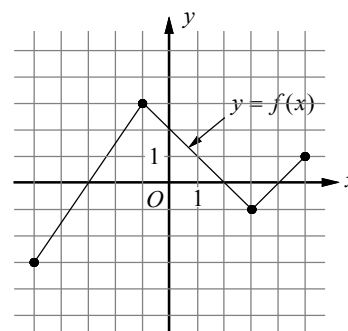
A function f is **decreasing** on an interval if the value of f decreases as x increases in the interval.

In the graph shown at the right, function f increases on the intervals $(-\infty, a)$ and (b, ∞) , and decreases on the interval (a, b) . At a point where the graph changes from increasing to decreasing, f has a local **maximum value**, and at a point where the graph changes from decreasing to increasing, f has a local **minimum value**.



Example 1 □ The complete graph of function f is shown at the right.

- Find the x -intercepts of $f(x)$.
- For what value of x is the value of $f(x)$ at its maximum?
- Find the interval where $f(x)$ is strictly decreasing.



- Solution** □
- The x -intercepts are -3 , 2 , and 4 .
 - The value of $f(x)$ is maximum at $x = -1$.
 - $f(x)$ is strictly decreasing between -1 and 3 .

Exercises - Polynomial Functions and Their Graphs

1

The graph of $f(x) = ax^3 + x^2 - 18x - 9$ intersects the x -axis at $(3, 0)$. What is the value of a ?

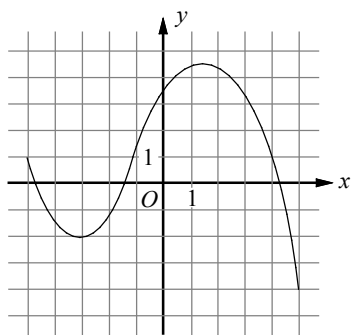
- A) -1
- B) 0
- C) 1
- D) 2

2

In the xy -plane, the graph of function f has x -intercepts at -7 , -5 , and 5 . Which of the following could define f ?

- A) $f(x) = (x-7)(x^2-25)$
- B) $f(x) = (x-7)(x^2+25)$
- C) $f(x) = (x+7)(x^2-25)$
- D) $f(x) = (x+7)(x^2+25)$

3

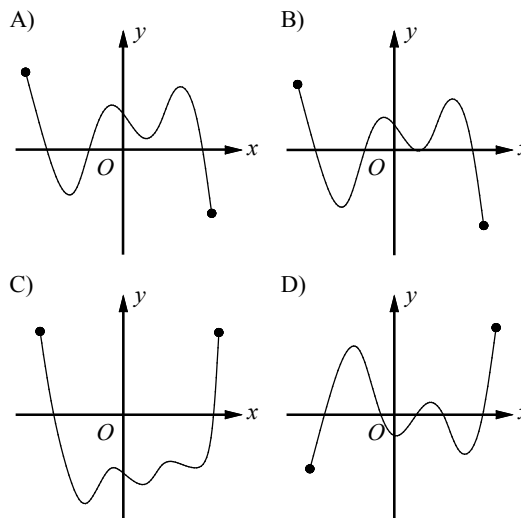


What is the minimum value of the function graphed on the xy -plane above, for $-5 \leq x \leq 5$?

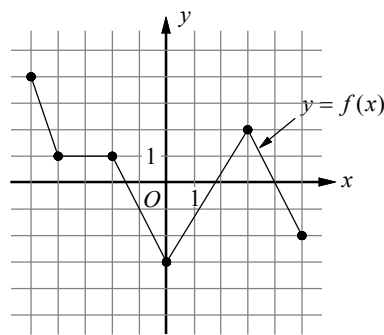
- A) -4
- B) -3
- C) -2
- D) $-\infty$

4

If function f has four distinct zeros, which of the following could represent the complete graph of f in the xy -plane?



5



The complete graph of function f is shown on the xy -plane above, for $-5 \leq x \leq 5$. Which of the following is/are true?

- I. f is strictly decreasing for $-5 < x < 0$.
- II. $f(-3) = 1$
- III. f is minimum at $x = 5$.

- A) I only
- B) II only
- C) III only
- D) I and II only

13-2. Remainder Theorem and Factor Theorem

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - c$, the remainder is $f(c)$.

Since Dividend = Divisor $\times$ Quotient + Remainder,

$f(x) = (x - c)q(x) + f(c)$, in which $q(x)$ is the quotient.

Factor Theorem

The polynomial $f(x)$ has $x - c$ as a factor if and only if $f(c) = 0$.

The following statements are equivalent for a polynomial $f(x)$ and a real number c :

- c is a **solution** to the equation $f(x) = 0$.
- c is a **zero** of $f(x)$.
- c is a **root** of $f(x)$.
- $x - c$ is a **factor** of $f(x)$.
- $f(x)$ is **divisible** by $x - c$.
- c is an **x -intercept** of the graph of $f(x)$.

Example 1 □ Find the remainder of $f(x) = x^3 + x^2 - 6x - 7$ divided by $x + 2$.

Solution □ To find the remainder of $f(x)$ divided by $x + 2 = x - (-2)$,
evaluate $f(-2)$. $f(-2) = (-2)^3 + (-2)^2 - 6(-2) - 7 = 1$
By the remainder theorem the remainder is 1.

Example 2 □ Find the value of a if $x - 3$ is a factor of $f(x) = x^3 - 11x + a$.

Solution □ If $x - 3$ is a factor of $f(x)$, then $f(3) = 0$
 $f(3) = (3)^3 - 11(3) + a = 0 \Rightarrow -6 + a = 0 \Rightarrow a = 6$

Example 3 □ Find the value of k if $f(x) = 3(x^2 + 3x - 4) - 8(x - k)$ is divisible by x .

Solution □ If $f(x)$ is divisible by x , since $x = x - 0$, $f(0) = 0$ by the factor theorem.
 $f(0) = 3(0^2 + 3(0) - 4) - 8(0 - k) = -12 + 8k = 0$
 $8k = 12 \Rightarrow k = \frac{12}{8} = \frac{3}{2}$

Example 4 □ Find the x -intercepts of f if $f(x) = 2x^2 + x - 10$.

Solution	□	$f(x) = 2x^2 + x - 10 = (2x + 5)(x - 2)$	Factor.
		$(2x + 5)(x - 2) = 0$	Let $f(x) = 0$.
		$2x + 5 = 0$ or $x - 2 = 0$	Zero Product Property
		$x = -\frac{5}{2}$ or $x = 2$	Solve.
		The x -intercepts of f are $-\frac{5}{2}$ and 2 .	

Exercises - Polynomial Functions and Their Graphs

1

If -1 and 1 are two real roots of the polynomial function $f(x) = ax^3 + bx^2 + cx + d$ and $(0, 3)$ is the y -intercept of graph of f , what is the value of b ?

- A) -3
- B) -1
- C) 2
- D) 4

2

What is the remainder of polynomial $p(x) = 81x^5 - 121x^3 - 36$ divided by $x + 1$?

- A) -76
- B) -36
- C) 4
- D) 6

3

If $x - 2$ is a factor of polynomial $p(x) = a(x^3 - 2x) + b(x^2 - 5)$, which of the following must be true?

- A) $a + b = 0$
- B) $2a - b = 0$
- C) $2a + b = 0$
- D) $4a - b = 0$

4

x	$f(x)$
-4	-10
-3	0
-1	-4
2	20

The function f is defined by a polynomial. Some values of x and $f(x)$ are shown in the table above. Which of the following must be a factor of $f(x)$?

- A) $x + 4$
- B) $x + 3$
- C) $x + 1$
- D) $x - 2$

5

$$x^3 - 8x^2 + 3x - 24 = 0$$

For what real value of x is the equation above true?

6

If $x > 0$, what is the solution to the equation $x^4 - 8x^2 = 9$?

13-3. Radical Expressions

The symbol $\sqrt[n]{a}$ is called a **radical**. Each part of a radical is given a name as indicated below.

$$\begin{array}{c} \text{radical sign} \\ \swarrow \quad \searrow \\ \text{index} \rightarrow \sqrt[n]{a} \leftarrow \text{radicand} \end{array}$$

Definition of n th root

For any real numbers x or a , and any positive integer n , if $x^n = a$, then x is an n th root of a .

If n is even, $x = \pm \sqrt[n]{a}$. If n is odd, $x = \sqrt[n]{a}$.

Definition of $a^{\frac{1}{2}}$ and $a^{\frac{1}{3}}$

For any nonnegative number a , $a^{\frac{1}{2}} = \sqrt{a}$. For any real number a , $a^{\frac{1}{3}} = \sqrt[3]{a}$.

Product and Quotient Property of Radicals

For any nonnegative number a or b , $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

For any real number a or b , $\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$ and $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$ if $b \neq 0$.

Example 1 □ Solve.

a. $(x-5)^4 = 16$

b. $x^3 + 1 = -26$

Solution □ a. $(x-5)^4 = 16$

$$x-5 = \pm \sqrt[4]{16}$$

$$x-5 = \pm 2$$

$$x = 5 \pm 2$$

$$x = 7 \text{ or } x = 3$$

Definition of n th root, for when n is even.

$$\sqrt[4]{16} = \sqrt[4]{2^4} = 2$$

Add 5 to each side.

Answer

b. $x^3 + 1 = -26$

$$x^3 = -27$$

$$x = \sqrt[3]{-27} = (-27)^{\frac{1}{3}} = ((-3)^3)^{\frac{1}{3}} = -3$$

Subtract 1 from each side.

Definition of n th root, for when n is odd.

Answer

Example 2 □ Simplify.

a. $\sqrt{50}\sqrt{6}$

b. $\sqrt{18a^2b^3}$

Solution □ a. $\sqrt{50}\sqrt{6} = \sqrt{25}\sqrt{2}\sqrt{2}\sqrt{3}$
 $= 5 \cdot 2 \cdot \sqrt{3} = 10\sqrt{3}$

$$\sqrt{50} = \sqrt{25}\sqrt{2}, \sqrt{6} = \sqrt{2}\sqrt{3}$$

b. $\sqrt{18a^2b^3} = \sqrt{3^2 \cdot 2 \cdot a^2 \cdot b^2 \cdot b}$
 $= \sqrt{3^2} \cdot \sqrt{2} \cdot \sqrt{a^2} \cdot \sqrt{b^2} \cdot \sqrt{b}$
 $= 3 \cdot \sqrt{2} \cdot a \cdot b \cdot \sqrt{b} = 3ab\sqrt{2b}$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{3^2} = 3, \sqrt{a^2} = a, \sqrt{b^2} = b$$

A method used to eliminate radicals from a denominator is called **rationalizing the denominator**.

Binomials of the form $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are called **conjugates**. The product of conjugates is always an integer if a and b are integers. You can use conjugates to rationalize denominators.

Adding and Subtracting Radical Expressions

Radical expressions in which the radicands are alike can be added or subtracted in the same way that like monomials are added or subtracted.

Multiplying Radical Expressions

Multiplying two radical expressions with different radicands is similar to multiplying binomials.

Example 3 □ Simplify.

$$\text{a. } \frac{1}{2 - \sqrt{3}}$$

$$\text{b. } (\sqrt{6} - \sqrt{2})(\sqrt{3} + 1)$$

$$\text{c. } \sqrt{50} - \sqrt{18} + \sqrt{8}$$

$$\text{d. } \sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}}$$

$$\begin{aligned} \text{Solution } \quad \text{a. } \frac{1}{2 - \sqrt{3}} &= \frac{1}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \\ &= \frac{2 + \sqrt{3}}{2^2 - (\sqrt{3})^2} \\ &= \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3} \end{aligned}$$

The conjugate of $2 - \sqrt{3}$ is $2 + \sqrt{3}$.

$$(a - b)(a + b) = a^2 - b^2$$

$$\begin{aligned} \text{b. } (\sqrt{6} - \sqrt{2})(\sqrt{3} + 1) \\ &= \sqrt{6} \cdot \sqrt{3} + \sqrt{6} \cdot 1 - \sqrt{2} \cdot \sqrt{3} - \sqrt{2} \cdot 1 \\ &= \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{3} + \sqrt{6} - \sqrt{6} - \sqrt{2} \\ &= 3\sqrt{2} - \sqrt{2} = 2\sqrt{2} \end{aligned}$$

FOIL

$$\sqrt{6} = \sqrt{2} \cdot \sqrt{3}$$

$$\begin{aligned} \text{c. } \sqrt{50} - \sqrt{18} + \sqrt{8} \\ &= \sqrt{25 \cdot 2} - \sqrt{9 \cdot 2} + \sqrt{4 \cdot 2} \\ &= \sqrt{25}\sqrt{2} - \sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{2} \\ &= 5\sqrt{2} - 3\sqrt{2} + 2\sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Combine like radicals.

$$\begin{aligned} \text{d. } \sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}} \\ &= \sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \sqrt{6} - \frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{2} \\ &= \sqrt{6} \left(1 - \frac{1}{3} + \frac{1}{2}\right) \\ &= \frac{7\sqrt{6}}{6} \end{aligned}$$

Rationalize the denominator.

Simplify.

Factor.

Exercises - Radical Expressions

1

Which of the following is equal to $a^{-\frac{1}{2}}$?

- A) $-\sqrt{a}$
- B) $\frac{1}{\sqrt{a}}$
- C) $-\frac{1}{\sqrt{a}}$
- D) $\frac{1}{a^2}$

2

Which of the following is equal to $\frac{1}{3-2\sqrt{2}}$?

- A) $3-\sqrt{2}$
- B) $3+\sqrt{2}$
- C) $3+2\sqrt{2}$
- D) $3+4\sqrt{2}$

3

If $(x+1)^3 = -64$, what is the value of x ?

- A) -6
- B) -5
- C) -4
- D) -3

4

Which of the following is equal to

$$\sqrt{8} + \sqrt{18} - \sqrt{32}?$$

- A) $\sqrt{2}$
- B) $2\sqrt{2}$
- C) $3\sqrt{2}$
- D) $\sqrt{3}$

5

Which of the following is equal to

$$(1+\sqrt{3})(2-\sqrt{3})?$$

- A) $1-\sqrt{3}$
- B) $1+\sqrt{3}$
- C) $-1-\sqrt{3}$
- D) $-1+\sqrt{3}$

6

Which of the following is equal to $b^{\frac{5}{3}}$?

- A) $b \cdot \sqrt{b}$
- B) $b \cdot \sqrt[3]{b^3}$
- C) $b \cdot \sqrt[3]{b}$
- D) $b \cdot \sqrt[3]{b^2}$

13-4. Solving Radical Equations

An equation which contains a radical with a variable in the radicand is called a **radical equation**. To solve such an equation, first isolate the radical on one side of the equation. Then square each side of the equation to eliminate the radical.

Example 1 □ Solve each equation.

a. $\sqrt{5-2x} = 3$

b. $4 + \sqrt{\frac{1}{2}x} = 7$

Solution □ a. $(\sqrt{5-2x})^2 = (3)^2$
 $5 - 2x = 9$
 $-2x = 4$
 $x = -2$

Square each side.

Subtract 5 from each side.

Divide each side by -2 .

b. $4 + \sqrt{\frac{1}{2}x} = 7$

Original Equation

$$\sqrt{\frac{1}{2}x} = 3$$

Subtract 3 from each side.

$$(\sqrt{\frac{1}{2}x})^2 = (3)^2$$

Square each side.

$$\frac{1}{2}x = 9$$

Simplify.

$$x = 18$$

Multiply each side by 2.

When you square both sides of a radical equation, the resulting equation may have a solution that is not a solution of the original equation. Such a solution is called an **extraneous solution**. Therefore, you must check all the possible solutions in the original equation and disregard the extraneous solutions.

Example 2 □ Solve $\sqrt{x+2} = x$.

Solution □ a. $\sqrt{x+2} = x$

Original equation

$$(\sqrt{x+2})^2 = (x)^2$$

Square each side.

$$x + 2 = x^2$$

Simplify.

$$0 = x^2 - x - 2$$

Subtract x and 2 from each side.

$$0 = (x-2)(x+1)$$

Factor.

$$x - 2 = 0 \text{ or } x + 1 = 0$$

Zero Product Property

$$x = 2 \text{ or } x = -1$$

Solve.

Check the results by substituting 2 and -1 for x in the original equation.

Check: $\sqrt{x+2} = x$

$$\sqrt{x+2} = x$$

$$\sqrt{2+2} = 2$$

$$\sqrt{-1+2} = -1$$

$$\sqrt{4} = 2$$

$$\sqrt{1} = -1$$

$$2 = 2 \quad \checkmark \quad \text{True}$$

$$\sqrt{1} \neq -1 \quad \times \quad \text{False}$$

Since -1 does not satisfy the original equation, 2 is the only solution.

Exercises - Solving Radical Equations

1

$$11 - \sqrt{2x + 3} = 8$$

What is the solution set of the equation above?

- A) 0
- B) 3
- C) 6
- D) 9

2

$$\sqrt{-3x + 4} = 7$$

What is the solution set of the equation above?

- A) -15
- B) -12
- C) -8
- D) -6

3

$$\sqrt{x + 18} = x - 2$$

What is the solution set of the equation above?

- A) $\{-2\}$
- B) $\{7\}$
- C) $\{-2, 7\}$
- D) $\{2, -7\}$

4

$$\sqrt{5x - 12} = 3\sqrt{2}$$

What is the solution set of the equation above?

- A) 2
- B) 4
- C) 6
- D) 8

5

If $a = \sqrt{3}$ and $\sqrt{2 - 3x} = \frac{1}{3}a$, what is the value of x ?

6

If $k = 8 - \sqrt{2}$ and $\sqrt[3]{x - k} = -2$, what is the value of x^2 ?

13-5. Complex Numbers

Definition of i

$$i = \sqrt{-1} \quad \text{and} \quad i^2 = -1$$

For real numbers a and b , the expression $a + bi$ is called a **complex number**.

Number a is called the **real part** and number b is called the **imaginary part** of the complex number $a + bi$.

To add or subtract complex numbers, combine the real parts and combine the imaginary parts.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Example 1 □ Simplify.

a. i^{35}

c. $(4 - 3i) + (5 + 4i)$

b. $\sqrt{-5} \cdot \sqrt{-10}$

d. $2(-3 + i) - 5(1 - i)$

Solution □ a. $i^{35} = i \cdot i^{34}$
 $= i \cdot (i^2)^{17}$
 $= i \cdot (-1)^{17}$
 $= -i$

b. $\sqrt{-5} \cdot \sqrt{-10}$
 $= (i \cdot \sqrt{5})(i \cdot \sqrt{10})$
 $= i^2 \sqrt{50}$
 $= (-1)(\sqrt{25} \cdot \sqrt{2})$
 $= -5\sqrt{2}$

c. $(4 - 3i) + (5 + 4i)$
 $= (4 + 5) + (-3i + 4i)$
 $= 9 + i$

d. $2(-3 + i) - 5(1 - i)$
 $= -6 + 2i - 5 + 5i$
 $= -11 + 7i$

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$i^2 = -1$$

$$(-1)^{17} = -1$$

$$\sqrt{-5} = \sqrt{5} \cdot \sqrt{-1} = i\sqrt{5}, \quad \sqrt{-10} = i\sqrt{10}$$

Multiply.

$$i^2 = -1, \quad \sqrt{50} = \sqrt{25} \cdot \sqrt{2}$$

Simplify.

Combine the real parts and the imaginary parts.

Simplify.

Multiply.

Simplify.

Example 2 □ Solve $3x^2 + 75 = 0$.

Solution □ $3x^2 + 75 = 0$

$$3x^2 = -75$$

$$x^2 = -25$$

$$x = \pm\sqrt{-25}$$

$$x = \pm\sqrt{25}\sqrt{-1}$$

$$x = \pm 5i$$

Subtract 75 from each side.

Divide each side by 3.

Take the square root of each side.

Product Property of Radicals

$$\sqrt{-1} = i$$

To multiply two complex numbers, use the FOIL method and use the fact that $i^2 = -1$.

$$(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$$

Imaginary numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**, and their product is the real number $a^2 + b^2$. This fact can be used to simplify the quotient of two imaginary numbers.

Definition of Equal Complex Numbers

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

$a + bi = c + di$ if and only if $a = c$ and $b = d$.

Example 3 □ Simplify.

a. $(6 - i)(2 + 3i)$

b. $(\sqrt{3} + \sqrt{-2})(\sqrt{3} - \sqrt{-2})$

c. $\frac{10}{(1 + 3i)}$

d. $\frac{2 + 3i}{4 - 3i}$

Solution □ a. $(6 - i)(2 + 3i)$

$$\begin{aligned} &= 12 + 18i - 2i - 3i^2 \\ &= 12 + 16i - 3(-1) \\ &= 15 + 16i \end{aligned}$$

FOIL
 $i^2 = -1$
Simplify.

b. $(\sqrt{3} - \sqrt{-2})(\sqrt{3} + \sqrt{-2})$
 $= (\sqrt{3} + i\sqrt{2})(\sqrt{3} - i\sqrt{2})$
 $= (\sqrt{3})^2 - (i\sqrt{2})^2$
 $= 3 - 2i^2$
 $= 5$

$\sqrt{-2} = i\sqrt{2}$
 $(a + b)(a - b) = a^2 - b^2$
 $i^2 = -1$

c. $\frac{10}{(1 + 3i)}$
 $= \frac{10}{(1 + 3i)} \cdot \frac{(1 - 3i)}{(1 - 3i)}$
 $= \frac{10(1 - 3i)}{1 - 9i^2}$
 $= \frac{\cancel{10}(1 - 3i)}{\cancel{10}}$
 $= 1 - 3i$

Rationalize the denominator.
 $(a + b)(a - b) = a^2 - b^2$
 $i^2 = -1$
 Simplify.

d. $\frac{2 + 3i}{4 - 3i}$
 $= \frac{2 + 3i}{4 - 3i} \cdot \frac{4 + 3i}{4 + 3i}$
 $= \frac{8 + 6i + 12i + 9i^2}{16 - 9i^2}$
 $= \frac{-1 + 18i}{25}$

Rationalize the denominator.
 FOIL
 $i^2 = -1$

Exercises - Complex Numbers

1

Which of the following is equal to $\sqrt{-1} - \sqrt{-4} + \sqrt{-9}$?

- A) i
- B) $2i$
- C) $3i$
- D) $4i$

2

Which of the following is equal to $\sqrt{-2} \cdot \sqrt{-8}$?

- A) $-4i$
- B) $4i$
- C) -4
- D) 4

3

Which of the following complex numbers is equal to $\frac{3-i}{3+i}$?

- A) $\frac{9}{10} - \frac{3i}{5}$
- B) $\frac{9}{10} + \frac{3i}{5}$
- C) $\frac{3}{5} - \frac{3i}{5}$
- D) $\frac{4}{5} - \frac{3i}{5}$

4

Which of the following is equal to $\frac{1}{2}(5i-3) - \frac{1}{3}(4i+5)$?

- A) $\frac{3}{2}i - \frac{5}{2}$
- B) $\frac{7}{6}i - \frac{7}{3}$
- C) $\frac{7}{6}i - \frac{19}{6}$
- D) $\frac{5}{6}i - \frac{17}{6}$

5

If $(4+i)^2 = a+bi$, what is the value of $a+b$?

6

If the expression $\frac{3-i}{1-2i}$ is rewritten in the form $a+bi$, in which a and b are real numbers, what is the value of $a+b$?

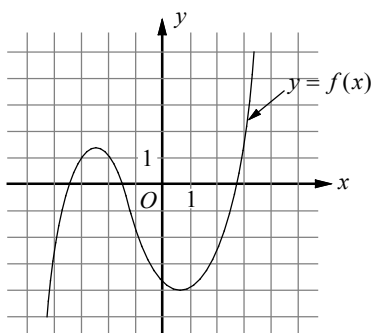
Chapter 13 Practice Test

1

If the graph of $f(x) = 2x^3 + bx^2 + 4x - 4$ intersects the x -axis at $(\frac{1}{2}, 0)$, and $(-2, k)$ lies on the graph of f , what is the value of k ?

- A) -4
- B) -2
- C) 0
- D) 2

2



The function $y = f(x)$ is graphed on the xy -plane above. If k is a constant such that the equation $f(x) = k$ has one real solution, which of the following could be the value of k ?

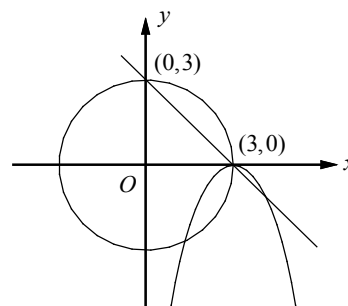
- A) -3
- B) -1
- C) 1
- D) 3

3

What is the value of a if $x + 2$ is a factor of $f(x) = -(x^3 + 3x^2) - 4(x - a)$?

- A) -2
- B) -1
- C) 0
- D) 1

4



$$x^2 + y^2 = 9$$

$$y = -(x - 3)^2$$

$$x + y = 3$$

A system of three equations and their graphs on the xy -plane are shown above. How many solutions does the system have?

- A) 1
- B) 2
- C) 3
- D) 4

5

Which of the following complex numbers is equivalent to $\frac{(1-i)^2}{1+i}$?

- A) $-\frac{i}{2} - \frac{1}{2}$
 B) $-\frac{i}{2} + \frac{1}{2}$
 C) $-i - 1$
 D) $-i + 1$

6

Which of the following is equal to $a\sqrt[3]{a}$?

- A) $a^{\frac{2}{3}}$
 B) $a^{\frac{4}{3}}$
 C) $a^{\frac{5}{3}}$
 D) $a^{\frac{7}{3}}$

7

$$p(x) = -2x^3 + 4x^2 - 10x$$

$$q(x) = x^2 - 2x + 5$$

The polynomials $p(x)$ and $q(x)$ are defined above. Which of the following polynomials is divisible by $x - 1$?

- A) $f(x) = p(x) - \frac{1}{2}q(x)$
 B) $g(x) = -\frac{1}{2}p(x) - q(x)$
 C) $h(x) = -p(x) + \frac{1}{2}q(x)$
 D) $k(x) = \frac{1}{2}p(x) + q(x)$

8

$$\sqrt{2x+6} = x+3$$

What is the solution set of the equation above?

- A) $\{-3\}$
 B) $\{-1\}$
 C) $\{-3, 2\}$
 D) $\{-3, -1\}$

9

What is the remainder when polynomial

$$p(x) = 24x^3 - 36x^2 + 14 \text{ is divided by } x - \frac{1}{2}?$$

- A) 4
 B) 6
 C) 8
 D) 10

10

The function f is defined by a polynomial. If $x+2$, $x+1$, and $x-1$ are factors of f , which of the following table could define f ?

A)

x	$f(x)$
-2	4
-1	0
1	0
2	0

B)

x	$f(x)$
-2	0
-1	4
1	0
2	0

C)

x	$f(x)$
-2	0
-1	0
1	4
2	0

D)

x	$f(x)$
-2	0
-1	0
1	0
2	4

Answer Key

Section 13-1

1. D 2. C 3. A 4. B 5. B

Section 13-2

1. A 2. C 3. D 4. B 5. 8

6. 3

Section 13-3

1. B 2. C 3. B 4. A 5. D

6. D

Section 13-4

1. B 2. A 3. B 4. C 5. $\frac{5}{9}$

6. 2

Section 13-5

1. B 2. C 3. D 4. C 5. 23

6. 2

Chapter 13 Practice Test

1. C 2. D 3. B 4. A 5. C

6. B 7. B 8. D 9. C 10. D

Answers and Explanations

Section 13-1

1. D

$$f(x) = ax^3 + x^2 - 18x - 9$$

If point $(3, 0)$ lies on the graph of f , substitute 0 for f and 3 for x .

$$0 = a(3)^3 + (3)^2 - 18(3) - 9.$$

$$0 = 27a - 54$$

$$2 = a$$

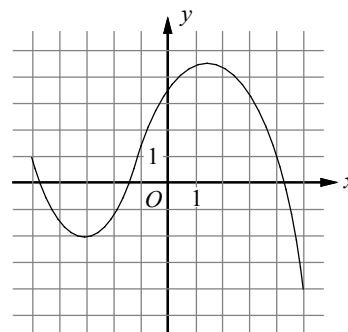
2. C

If the graph of a polynomial function f has an x -intercept at a , then $(x - a)$ is a factor of $f(x)$.

Since the graph of function f has x -intercepts at -7 , -5 , and 5 , $(x + 7)$, $(x + 5)$, and $(x - 5)$ must each be a factor of $f(x)$. Therefore,

$$f(x) = (x + 7)(x + 5)(x - 5) = (x + 7)(x^2 - 5).$$

3. A



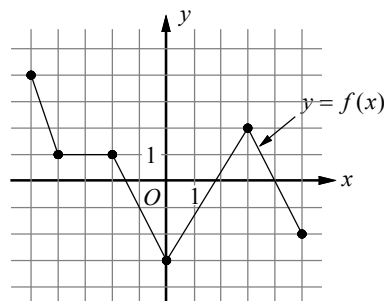
The minimum value of a graphed function is the minimum y -value of all the points on the graph. For the graph shown, when $x = -3$, $y = -2$ and when $x = 5$, $y = -4$, so the minimum is at $(5, -4)$ and the minimum value is -4 .

4. B

A zero of a function corresponds to an x -intercept of the graph of the function on the xy -plane.

Only the graph in choice B has four x -intercepts. Therefore, it has the four distinct zeros of function f .

5. B



I. f is not strictly decreasing for $-5 < x < 0$, because on the interval $-4 < x < -2$, f is not decreasing.

Roman numeral I is not true.

II. The coordinates $(-3, 1)$ is on the graph of f , therefore, $f(-3) = 1$

Roman numeral II is true.

III. For the graph shown, when $x = 0$, $y = -3$ and when $x = 5$, $y = -2$, so f is minimum at $x = 0$.

Roman numeral III is not true.

Section 13-2

1. A

If -1 and 1 are two real roots of the polynomial function, then $f(-1) = 0$ and $f(1) = 0$. Thus

$$f(-1) = a(-1)^3 + b(-1)^2 + c(-1) + d = 0 \text{ and}$$

$$f(1) = a(1)^3 + b(1)^2 + c(1) + d = 0.$$

Simplify the two equations and add them to each other.

$$-a + b - c + d = 0$$

$$+ \underline{a + b + c + d = 0}$$

$$2b + 2d = 0 \text{ or } b + d = 0.$$

Also $f(0) = 3$, since the graph of the polynomial passes through $(0, 3)$.

$$f(0) = a(0)^3 + b(0)^2 + c(0) + d = 3 \text{ implies } d = 3.$$

Substituting $d = 3$ in the equation $b + d = 0$ gives $b + 3 = 0$, or $b = -3$.

2. C

If polynomial $p(x) = 81x^5 - 121x^3 - 36$ is divided by $x + 1$, the remainder is $p(-1)$.

$$p(-1) = 81(-1)^5 - 121(-1)^3 - 36 = 4$$

The remainder is 4.

3. D

If $x - 2$ is a factor for polynomial $p(x)$, then

$$p(2) = 0.$$

$$p(x) = a(x^3 - 2x) + b(x^2 - 5)$$

$$p(2) = a(2^3 - 2(2)) + b(2^2 - 5)$$

$$= a(8 - 4) + b(4 - 5)$$

$$= 4a - b = 0$$

4. B

If $(x - a)$ is a factor of $f(x)$, then $f(a)$ must be equal to 0. Based on the table, $f(-3) = 0$.

Therefore, $x + 3$ must be a factor of $f(x)$.

5. 8

$$x^3 - 8x^2 + 3x - 24 = 0$$

$$(x^3 - 8x^2) + (3x - 24) = 0 \quad \text{Group terms.}$$

$$x^2(x - 8) + 3(x - 8) = 0 \quad \text{Factor out the GCF.}$$

$$(x^2 + 3)(x - 8) = 0 \quad \text{Distributive Property}$$

$$x^2 + 3 = 0 \text{ or } x - 8 = 0 \quad \text{Solutions}$$

Since $x^2 + 3 = 0$ does not have a real solution, $x - 8 = 0$, or $x = 8$, is the only solution that makes the equation true.

6. 3

$$x^4 - 8x^2 = 9$$

$$x^4 - 8x^2 - 9 = 0 \quad \text{Make one side 0.}$$

$$(x^2 - 9)(x^2 + 1) = 0 \quad \text{Factor.}$$

$$(x + 3)(x - 3)(x^2 + 1) = 0 \quad \text{Factor.}$$

Since $x^2 + 1 = 0$ does not have a real solution, the solutions for x are $x = -3$ and $x = 3$.

Since it is given that $x > 0$, $x = 3$ is the only solution to the equation.

Section 13-3

1. B

$$a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{a}}$$

2. C

$$\frac{1}{3 - 2\sqrt{2}}$$

$$= \frac{1}{3 - 2\sqrt{2}} \cdot \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}} \quad \text{Multiply the conjugate of the denominator.}$$

$$= \frac{3 + 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2} \quad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{3 + 2\sqrt{2}}{9 - 8} \quad \text{Simplify.}$$

$$= 3 + 2\sqrt{2}$$

3. B

$$(x + 1)^3 = -64$$

$$x + 1 = \sqrt[3]{-64} \quad \text{Definition of cube root.}$$

$$x + 1 = -4 \quad \sqrt[3]{-64} = (-64)^{\frac{1}{3}} = -4$$

$$x = -5 \quad \text{Subtract 1 from each side.}$$

4. A

$$\begin{aligned} & \sqrt{8} + \sqrt{18} - \sqrt{32} \\ &= \sqrt{4}\sqrt{2} + \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{2} \\ &= 2\sqrt{2} + 3\sqrt{2} - 4\sqrt{2} \\ &= \sqrt{2} \end{aligned}$$

5. D

$$\begin{aligned}
 (1+\sqrt{3})(2-\sqrt{3}) &= 2-\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3} && \text{FOIL} \\
 &= 2+\sqrt{3}-3 && \text{Combine like radicals.} \\
 &= -1+\sqrt{3} && \text{Simplify.}
 \end{aligned}$$

6. D

$$b^{\frac{5}{3}} = b^1 \cdot b^{\frac{2}{3}} = b \cdot (b^2)^{\frac{1}{3}} = b \cdot \sqrt[3]{b^2}$$

Section 13-4

1. B

$$\begin{aligned}
 11-\sqrt{2x+3} &= 8 \\
 11-\sqrt{2x+3}-11 &= 8-11 && \text{Subtract 11 from each side.} \\
 -\sqrt{2x+3} &= -3 && \text{Simplify.} \\
 (-\sqrt{2x+3})^2 &= (-3)^2 && \text{Square each side.} \\
 2x+3 &= 9 && \text{Simplify.} \\
 2x &= 6 && \text{Subtract 3 from each side.} \\
 x &= 3 && \text{Divide each side by 2.}
 \end{aligned}$$

2. A

$$\begin{aligned}
 \sqrt{-3x+4} &= 7 \\
 (\sqrt{-3x+4})^2 &= (7)^2 && \text{Square each side.} \\
 -3x+4 &= 49 && \text{Simplify.} \\
 -3x &= 45 && \text{Subtract 4 from each side.} \\
 x &= -15 && \text{Divide each side by } -3.
 \end{aligned}$$

3. B

$$\begin{aligned}
 \sqrt{x+18} &= x-2 \\
 (\sqrt{x+18})^2 &= (x-2)^2 && \text{Square each side.} \\
 x+18 &= x^2-4x+4 && \text{Simplify.} \\
 0 &= x^2-5x-14 && \text{Make one side 0.} \\
 0 &= (x-7)(x+2) && \text{Factor.} \\
 0 &= x-7 \text{ or } 0 = x+2 && \text{Zero Product Property} \\
 7 &= x \text{ or } -2 = x
 \end{aligned}$$

Check each x -value in the original equation.

$$\begin{aligned}
 \sqrt{7+18} &= 7-2 && x=7 \\
 \sqrt{25} &= 5 && \text{Simplify.} \\
 5 &= 5 && \text{True} \\
 \sqrt{-2+18} &= -2-2 && x=-2 \\
 \sqrt{16} &= -4 && \text{Simplify.} \\
 4 &= -4 && \text{False}
 \end{aligned}$$

Thus, 7 is the only solution.

4. C

$$\begin{aligned}
 \sqrt{5x-12} &= 3\sqrt{2} \\
 (\sqrt{5x-12})^2 &= (3\sqrt{2})^2 && \text{Square each side.} \\
 5x-12 &= 18 && \text{Simplify.} \\
 5x &= 30 && \text{Add 12 to each side.} \\
 x &= 6 && \text{Divide by 5 on each side.}
 \end{aligned}$$

5. $\frac{5}{9}$

$$\begin{aligned}
 \sqrt{2-3x} &= \frac{1}{3}a \\
 \sqrt{2-3x} &= \frac{1}{3}\sqrt{3} && a=\sqrt{3} \\
 (\sqrt{2-3x})^2 &= \left(\frac{1}{3}\sqrt{3}\right)^2 && \text{Square each side.} \\
 2-3x &= \frac{1}{3} && \text{Simplify.} \\
 -3x &= -\frac{5}{3} && \text{Subtract 2 from each side.} \\
 -\frac{1}{3}(-3x) &= -\frac{1}{3}\left(-\frac{5}{3}\right) && \text{Multiply each side by } -\frac{1}{3}. \\
 x &= \frac{5}{9} && \text{Simplify.}
 \end{aligned}$$

6. 2

$$\begin{aligned}
 \sqrt[3]{x-k} &= -2 \\
 (\sqrt[3]{x-k})^3 &= (-2)^3 && \text{Cube each side.} \\
 x-k &= -8 && \text{Simplify.} \\
 x-(8-\sqrt{2}) &= -8 && k=8-\sqrt{2} \\
 x-8+\sqrt{2} &= -8 && \text{Simplify.} \\
 x+\sqrt{2} &= 0 && \text{Add 8 to each side.} \\
 x &= -\sqrt{2} && \text{Subtract } \sqrt{2}. \\
 (x)^2 &= (-\sqrt{2})^2 && \text{Square each side.} \\
 x^2 &= 2 && \text{Simplify.}
 \end{aligned}$$

Section 13-5

1. B

$$\begin{aligned}
 \sqrt{-1}-\sqrt{-4}+\sqrt{-9} &= i-i\sqrt{4}+i\sqrt{9} && i=\sqrt{-1} \\
 &= i-2i+3i \\
 &= 2i
 \end{aligned}$$

2. C

$$\begin{aligned}
 &\sqrt{-2} \cdot \sqrt{-8} \\
 &= i\sqrt{2} \cdot i\sqrt{8} & \sqrt{-2} = i\sqrt{2}, \sqrt{-8} = i\sqrt{8} \\
 &= i^2 \sqrt{16} \\
 &= -4 & i^2 = -1
 \end{aligned}$$

3. D

$$\begin{aligned}
 &\frac{3-i}{3+i} \\
 &= \frac{3-i}{3+i} \cdot \frac{3-i}{3-i} & \text{Rationalize the denominator.} \\
 &= \frac{9-6i+i^2}{9-i^2} & \text{FOIL} \\
 &= \frac{9-6i-1}{9+1} & i^2 = -1 \\
 &= \frac{8-6i}{10} & \text{Simplify.} \\
 &= \frac{4-3i}{5} \text{ or } \frac{4}{5} - \frac{3i}{5}
 \end{aligned}$$

4. C

$$\begin{aligned}
 &\frac{1}{2}(5i-3) - \frac{1}{3}(4i+5) \\
 &= \frac{5}{2}i - \frac{3}{2} - \frac{4i}{3} - \frac{5}{3} & \text{Distributive Property} \\
 &= \frac{15}{6}i - \frac{9}{6} - \frac{8i}{6} - \frac{10}{6} & 6 \text{ is the GCD.} \\
 &= \frac{7}{6}i - \frac{19}{6} & \text{Simplify.}
 \end{aligned}$$

5. 23

$$\begin{aligned}
 (4+i)^2 &= a+bi \\
 16+8i+i^2 &= a+bi & \text{FOIL} \\
 16+8i-1 &= a+bi & i^2 = -1 \\
 15+8i &= a+bi & \text{Simplify.} \\
 15 = a \text{ and } 8 = b & & \text{Definition of Equal Complex Numbers}
 \end{aligned}$$

Therefore, $a+b=15+8=23$.

6. 2

$$\begin{aligned}
 \frac{3-i}{1-2i} &= \frac{3-i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{3+6i-i-2i^2}{1-4i^2} \\
 &= \frac{3+6i-i+2}{1+4} = \frac{5+5i}{5} = 1+i = a+bi
 \end{aligned}$$

Therefore, $a=1$ and $b=1$, and $a+b=1+1=2$.**Chapter 13 Practice Test**

1. C

$$f(x) = 2x^3 + bx^2 + 4x - 4$$

$f\left(\frac{1}{2}\right) = 0$ because the graph of f intersects the x -axis at $\left(\frac{1}{2}, 0\right)$.

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + b\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 4 = 0$$

Solving the equation for b gives $b=7$.

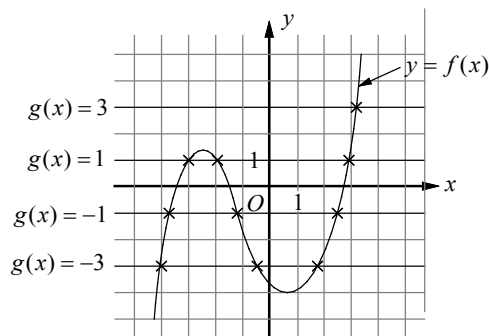
$$\text{Thus } f(x) = 2x^3 + 7x^2 + 4x - 4.$$

Also $k = f(-2)$, because $(-2, k)$ lies on the graph of f .

$$k = f(-2) = 2(-2)^3 + 7(-2)^2 + 4(-2) - 4$$

Solving the equation for k gives $k=0$.

2. D



$g(x) = -3$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = -1$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = 1$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = 3$ has 1 point of intersection with $y = f(x)$, so there is 1 real solution.

Choice D is correct

3. B

If $x+2$ is a factor of

$$f(x) = -(x^3 + 3x^2) - 4(x-a), \text{ then } f(-2) = 0.$$

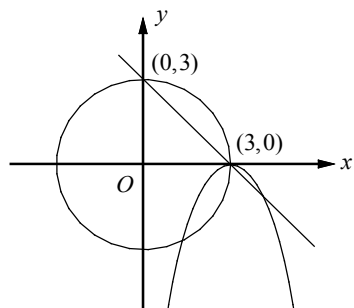
$$f(-2) = -((-2)^3 + 3(-2)^2) - 4(-2-a) = 0$$

$$-(-8+12)+8+4a=0$$

$$4+4a=0$$

$$a=-1$$

4. A



The solutions to the system of equations are the points where the circle, parabola, and line all intersect. That point is $(3, 0)$ and is therefore the only solution to the system.

5. C

$$\begin{aligned} \frac{(1-i)^2}{1+i} &= \frac{1-2i+i^2}{1+i} && \text{FOIL the numerator.} \\ &= \frac{1-2i-1}{1+i} && i^2 = -1 \\ &= \frac{-2i}{1+i} && \text{Simplify.} \\ &= \frac{-2i}{1+i} \cdot \frac{1-i}{1-i} && \text{Rationalize the denominator.} \\ &= \frac{-2i+2i^2}{1-i^2} && \text{FOIL} \\ &= \frac{-2i-2}{2} && i^2 = -1 \\ &= -i-1 \end{aligned}$$

6. B

$$a \sqrt[3]{a} = a \cdot a^{\frac{1}{3}} = a^{1+\frac{1}{3}} = a^{\frac{4}{3}}$$

7. B

$$p(x) = -2x^3 + 4x^2 - 10x$$

$$q(x) = x^2 - 2x + 5$$

In $p(x)$, factoring out the GCF, $-2x$, yields

$$p(x) = -2x(x^2 - 2x + 5) = -2x \cdot q(x).$$

Let's check each answer choice.

$$\begin{aligned} \text{A) } f(x) &= p(x) - \frac{1}{2}q(x) \\ &= -2x \cdot q(x) - \frac{1}{2}q(x) = (-2x - \frac{1}{2})q(x) \end{aligned}$$

$q(x)$ is not a factor of $x-1$ and $(-2x - \frac{1}{2})$ is not a factor of $x-1$. $f(x)$ is not divisible by $x-1$.

$$\begin{aligned} \text{B) } g(x) &= -\frac{1}{2}p(x) - q(x) \\ &= -\frac{1}{2}[-2x \cdot q(x)] - q(x) = (x-1)q(x) \end{aligned}$$

Since $g(x)$ is $x-1$ times $q(x)$, $g(x)$ is divisible by $x-1$.

Choices C and D are incorrect because $x-1$ is not a factor of the polynomials $h(x)$ and $k(x)$.

8. D

$$\begin{aligned} \sqrt{2x+6} &= x+3 \\ (\sqrt{2x+6})^2 &= (x+3)^2 && \text{Square each side.} \\ 2x+6 &= x^2+6x+9 && \text{Simplify.} \\ x^2+4x+3 &= 0 && \text{Make one side 0.} \\ (x+1)(x+3) &= 0 && \text{Factor.} \\ x+1=0 \text{ or } x+3=0 &&& \text{Zero Product Property} \\ x=-1 \text{ or } x=-3 \end{aligned}$$

Check each x -value in the original equation.

$$\begin{aligned} \sqrt{2(-1)+6} &= -1+3 && x=-1 \\ \sqrt{4} &= 2 && \text{Simplify.} \\ 2 &= 2 && \text{True} \end{aligned}$$

$$\begin{aligned} \sqrt{2(-3)+6} &= -3+3 && x=-3 \\ 0 &= 0 && \text{True} \end{aligned}$$

Thus, -1 and -3 are both solutions to the equation.

9. C

Use the remainder theorem.

$$p\left(\frac{1}{2}\right) = 24\left(\frac{1}{2}\right)^3 - 36\left(\frac{1}{2}\right)^2 + 14 = 8$$

Therefore, the remainder of polynomial

$$p(x) = 24x^3 - 36x^2 + 14 \text{ divided by } x - \frac{1}{2} \text{ is } 8.$$

10. D

If $(x-a)$ is a factor of $f(x)$, then $f(a)$ must equal to 0. Thus, if $x+2$, $x+1$ and $x-1$ are factors of f , we have $f(-2) = f(-1) = f(1) = 0$.

Choice D is correct.